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Differences in growth of Sindhi and crosses of Sindhi with Jersey, Brown Swiss, Holstein-Friesian and Guernsey cattle

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DIFFERENCES IN GROWTH OF SINDHI AND CROSSES OF SINDHI WITH JERSEY, BROWN SWISS, HOLSTEIN-FRIESIAN AND GUERNSEY CATTLE

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INTRODUCTION

Man's quest for improvement is eternal. It covers all fields of human activity and endeavour. In the improvement of domestic animals, conscious efforts have probably been going on since domestication began. The history of domestication and of the beginnings of systematic livestock improvement can be found in most text books on animal breeding.

Introduction of new types of livestock was an important part of these efforts in individual regions. Early attempts of this nature were more or less localized for the want of a well developed system of transportion. Invaders and immigrants often carried some of their own livestock to the new lands. This stock was then interbred with the existing animals in the land of conquest or immigration, or was maintained as a distinct group. The results of such introductions have varied. In general, the introduction of farm livestock in virgin areas has been successful. Introduction of new types of livestock into areas where well adapted indigenous stock already existed has not always been of unqualified benefit. To this category belongs the introduction of European breeds of cattle into India.

Numerous organized and unorganized efforts have been made to improve local Indian stock by introducing several breeds of European cattle. Due to various reasons connected with economic, climatic and disease conditions, herds of pure bred European cattle have not been established in India on any sizable scale. The introduction of European cattle generally took the form of importing bulls and resorting to "cross breeding". The term, cross breeding, has acquired a special meaning in India (MacGuckin, 1937). It refers almost exclusively to breeding of indigenous stock to European. Thus breeding of Sindhi animals to Jersey, Holstein-Friesian or any other breed of European cattle is called "cross breeding"; while interbreeding of two breeds of Indian cattle is not always termed such.

The main purpose of introducing European breeds of cattle, or of "cross breeding", has been to increase the milk yield. Although some individual cows of Indian breeds in certain herds have produced well over 10,000 pounds of milk in a year (Olver, 1934; Sayer, 1936), the average yield of Indian cows remains low. The annual milk yield has been estimated at 600 pounds per cow (Wright, 1937) for the whole country, and 943 pounds per cow for certain cattle breeding areas (Olver, 1937, a). In a later report (Anonymous, 1942), the estimated annual average yields are stated to range from 65 pounds to 1442 pounds in different regions with an all-India average of 487 pounds per cow. These figures undoubtedly provide adequate reason for attempts to increase the milk production. In recent years, particularly after the publication of the Report of the Royal Commission on Agriculture in 1928, "cross breeding" as a method of improving milk yields of Indian cattle has met with disfavor from government officials. Colonel Sir Arthur Olver (1937, b, p. 463), Animal Husbandry Expert, Imperial Council of Agricultural Research, stated:

> For years past it has been demonstrated by organizations such as the Military Dairy Farms that cross-bred cattle from Indian cows by sires of European blood, in spite of heavy capital and recurring expenditure involved, are generally, under their special conditions of management, more profitable dairy animals than ordinary Indian cows. On the other hand, there is ample evidence to show that where control is inadequate or inexpert, the pursuit of such a policy leads to immediate loss of type, rapid degeneration and high susceptibility to disease.

The same author expressed his opinion in a later statement (Olver, 1938, p. 493):

> The improvement of livestock and of cattle in particular, is indeed a matter of great social as well as economic importance and urgency in India and it is necessary to emphasize that it has now been amply shown that it is unnecessary and unsound as a general policy to attempt to introduce European breeds of cattle into India.

The adaptability of the new stock to the environments of the region where it is introduced is important. Hagedoorn (1946) has amply emphasized this. The problem of the suitability of a particular European breed of cattle for introduction into India has not been extensively studied. Many characters such as resistance to disease, ability to withstand climatic conditions, adaptability to a comparatively low plane of husbandry and the like, must be studied before this question can be settled.

Growth, whether measured in weight or in linear dimensions, is one such attribute which reflects the suitability of a breed or strain to a particular environment. It is probably not so directly important for dairy cattle as for animals used for producing meat. Its importance as an index of constitution, vigor, maturity and general desirability is clear, however. In this study an attempt has been made to evaluate the differences in growth of Sindhi and crosses of Sindhi with Jersey, Brown Swiss, Holstein-Friesian and Guernsey breeds of cattle.

REVIEW OF LITERATURE

Growth of European Breeds of Dairy Cattle

Eckles initiated investigations on growth of dairy cattle in 1906 (Brody and Ragsdale 1921). Out of this project have come many reports by various workers on different phases of growth in dairy cattle. Eckles (1915) studied the effect of feeding and age of calving on the growth of dairy heifers. He concluded that heavy feeding resulted in rapid growth but the light fed animals continued to grow for a longer period. He also observed that the difference in feeding had more effect upon the weight than on the skeleton and that gestation did not hinder growth while lactation did.

Eckles and Swett (1918) studied the influence of birth weight, breed, gestation, plane of nutrition and similar factors on the growth of dairy heifers. They concluded that weight at birth and pregnancy did not influence the growth of dairy cattle and that the plane of nutrition and lactation had important effects. They found that the growth of Jerseys and Holsteins was similar up to 24 months of age. After that, the Holsteins grew at a faster rate. The authors found breed differences in the age at maturity, Jersey animals reaching skeletal maturity between three and four years while the Holstein animals did so between four and five years. Eckles (1920) presented growth curves for weight and height based on monthly observations on Jersey, Holstein-Friesian, Ayrshire and dairy Shorthorn cows at the University of Missouri farm.

Turner, et al. (1923) studied the growth of cows in the Jersey Register of Merit, from one and a half to 17 years of age. Brody and Ragsdale (1923) presented these data together with those from the University of Missouri farm and formulated a curve for the relation between height at withers and body weight for Holstein and Jersey cows. The authors postulated that the weight-height relationship was the same for all cows. Ragsdale et al. (1926) added data from the United States Department of Agriculture and the Kansas Agricultural Experiment Station to those from the University of Missouri farm. They also included information on growth of male calves at the University of Missouri farm. The authors presented data on prenatal and post-natal growth and postulated that a "break" occurred in the prenatal growth at about six months of pregnancy and in post-natal growth at about four months of age. The authors raised the question as to what was to be considered "normal" growth. Ragsdale and Regan (1930) studied measurements of Jersey and Holstein cattle on Missouri farms and presented prediction charts for growth of cattle.

Ashton (1930) measured over 1500 cattle of Brown Swiss, Brittany, Lombardy, Beef Shorthorn, Dairy Shorthorn and Ayrshire breeds in Europe and the British islands. He presented information on the growth of these animals. Bartlett and Jameson (1931) studied the growth of Milking Shorthorn animals at Reading, England, and constructed a table for normal growth of cows up to five years of age.

Espe et al. (1932) studied the growth of cattle at Iowa State College dairy farm and made comparisons with data published elsewhere. Ragsdale (1934) combined the existing information from Iowa, Kansas and Missouri and presented growth standards for dairy cattle. Morrison (1946) summarized available information on weights and heights of males and females of the Ayrshire, Guernsey, Jersey and Holstein breeds and presented tables of normal growth from birth to two years of age. He used data from Iowa, Kansas, Missouri, Nebraska, South Carolina and West Virginia Agricultural Experiment Stations and from the United States Department of Agriculture. The numbers on which the averages were based varied, being larger for earlier ages. The averages for height were based on smaller numbers than those for weight and the number of males included was smaller than the number of females.

Growth of Indian Cattle

Reports on growth of cattle in India are not numerous. Several reports mentioning birth weights of various breeds of cattle have been published. Sayer (1934) reported growth of 63 Sahiwal calves. These were divided into two groups. One group of 30 calves was under special feeding and the other group of 33 calves was under ordinary feeding. He reported weekly weights of these animals from birth to 25 weeks of age. He also reported weekly weights of 12 specially fed calves from birth to 52 weeks of age. He fitted a linear regression and a parabola to the data for 63 calves (birth to 26 weeks) and concluded that the growth of ordinarily fed calves was more uniform than that of specially fed calves. He found no significant differences in growth of the two groups for the first 19 weeks, after which the special group gained more.

Ayyar (1935) studied the weights of the same 63 Sahiwal calves reported by Sayer (1934). He found a correlation of 0.9936 between the weight and the age (birth to 26 weeks) for the special calves and 0.9966 for the ordinary calves. These correlations apparently concerned weekly mean weights. The author computed linear regressions, and concluded that the growth rates of the two groups of calves were different. The linear regressions fitted poorly, however. The author also attempted to fit logarithmic lines and, finding them unsatisfactory, fitted parabolic curves to the data.

Sayer (1936) reported weights of Sahiwal cattle in the Pusa herd under two systems of feeding and management; one called ordinary and the other designed to promote early maturity. In this report, he presented individual weekly weights of eight calves in each group from birth to 22 weeks. He also presented weights of 37 females and 27 males at birth, at 24 weeks and at 40 weeks. He attempted to show that the animals fed for early maturity grew faster than those fed under the ordinary system. A comparison with the weekly weights reported by the author earlier (Sayer, 1934) shows that the weights of calves in both groups reported in 1936 were little lower than those reported in 1934. The differences between the two groups were of about the same magnitude in both reports.

Growth of Crosses Between Indian and European Cattle

Lush *et al.* (1930) studied the growth of high grade Herefords, half-blood Brahmans and quarter-blood Brahmans on range in Texas. The animals called half-blood and quarter-blood Brahmans were obtained by the use of three different bulls, none of which was a pure bred; one was three-quarter blood Brahman, another was sixty-one sixty-fourths and the third was about fifteen-sixteenths Brahman. The term, Brahman, is not explained but probably applied in this case to animals of Nellore and Krishna Valley breeding. At any rate they were descended from cattle of the heavy draft type which had been imported from India in 1906 or earlier (Lush, 1949). The authors found that at about 30 months the quarter-blood Brahmans were slightly heavier than the half-blood Brahmans or high grade Herefords. It was concluded that the breed differences were relatively unimportant as compared with differences traceable to the condition of pastures.

Littlewood (1933) reported on cross breeding between Sindhi and Ayrshire in Madras. In this project, the first generation half bred bulls were used on the first generation cows, the second generation bulls on the second generation cows and so on. In this way the strain retained half the imported blood and half the native blood. He presented the birth weight of males and females and the weight, height at withers and heart girth of cows at four years (Table 1).

Table 1. Weight at birth and weight, height and heart girth at four years for Ayrshire-Sindhi crosses (Littlewood, 1933).

	Weight at birth		Females at four years		
	Male (pounds)	Female (pounds)	Weight (pounds)	Height (inches)	Girth (inches)
F_1	52.0	51.0	859	47.5	67.5
F_2	59.0	49.0	750	46.5	65.0
F_3	54.0	46.0	701	45.0	63.5
F_4	49.0	48.0	755	43.5	61.0
F_5	-	48.5	-	-	-

He (Littlewood, 1933, p. 327) stated:

> The calves of the second, third, etc. generations do not grow so well as the F_1 calves; this, no doubt, is partly due to hand feeding ....

Regarding the mature weights, this author (Littlewood, 1933, p. 331) stated:

> Both height and girth are decreasing in each generation and the animals appear to be reverting more towards the size of the Sindhi dam. The highest weight of an F_1 cow is 952 lbs. and the lowest weight is 776 lbs., the largest cow standing 51 inches and the smallest 45 inches. In F_2 cows the heaviest cow is 894 lbs. and the smallest is 672 lbs., the heights being 51 inches and 44 inches. In F_3 the heaviest cow is 796 lbs. and the smallest 576 lbs., the heights being $47\frac{1}{2}$ inches and $43\frac{1}{2}$ inches.

Schutte (1935) studied the growth of crosses between local "unimproved" stock and five breeds of cattle under range conditions in a semi-arid area in South Africa. In this experiment, bulls of Hereford, Shorthorn, Sussex, Aberdeen-Angus and Africander breeds were used on cows of miscellaneous ancestry and the growth of the half bred progeny was studied. The author studied weights and a number of body measurements and found that the growth from weaning to maturity was seasonal.

He also found that the crosses with Hereford and with Sussex exceeded the crosses with Aberdeen-Angus and Africander in weights and most body measurements, while the crosses with Shorthorns occupied an intermediate position. Howe (1946) studied the growth of various crosses of Sahiwal with Jersey, Guernsey and Holstein-Friesian cattle in Jamaica. He concluded that animals with Sahiwal blood had higher birth weights and grew more rapidly than purebred European cattle. He found that the increase was proportional to the amount of Sahiwal blood. In this study Howe did not have any animal having more than 50 per cent Sahiwal blood.

THE INVESTIGATION

Material

Location of the herd

The data used in this study were obtained from records kept at the Allahabad Agriculture Institute, Allahabad (United Provinces), India. Allahabad is located at longitude 81° 44' E., latitude 25° 27' N., at an elevation of 322 feet above mean sea level. The annual mean minimum temperature is 66.4° F., and the mean maximum temperature is 90.1°F. The average annual rainfall is 41.8 inches (India Meteorological Department, 1949). The winter temperatures reach lows of 32-34° F. and the summer temperatures go as high as 118-120° F. The rainfall is received mainly in July, August and September.

A brief history of the herd

A herd of cattle was started at Allahabad Agricultural Institute in 1910. Several breeds of Indian cattle as well as their crosses with four breeds of European cattle were used. Originally, cows of Hissar and Gir breeds were kept. Some Sahiwal cows were purchased in 1916 and the first Sindhi cows were obtained in 1917 (Higginbottom, 1949). In 1923, 32 Sindhi cows and one Sindhi bull were purchased. In 1929, 12 Sindhi cows, 12 Kankrej cows and eight Hariana cows were bought. Sixteen Sindhi cows were obtained in 1932. Four Sindhi bulls were obtained in 1935 from the herd kept at the Agricultural College, Poona, India (Schneider, 1949). And 16 Sindhi cows were purchased in the same year. The last purchase of Sindhi animals was in 1946 when 12 cows were obtained (Joshi, 1949).

A consignment of two Jersey bulls, one Brown Swiss bull, one Holstein-Friesian bull and one Guernsey bull, donated by various individuals in the United States of America, was received in 1921. All these animals died during the process of immunization against Rinderpest in November of the same year. On hearing of the situation, donors of the first lot offered to repeat their gifts. A second set of five bulls reached Bombay in the fall of 1922. One of these was killed during unloading from the ship. The remaining four reached Allahabad safely and remained in service in the herd for varying periods of time (Higginbottom, 1949). In 1925, four bulls, two Jersey, one Brown Swiss and one Holstein-Friesian, were imported. Still another consignment of five bulls, one Jersey, one Brown Swiss, two Holstein-Friesian and one Guernsey was received in the winter of 1926 (Joshi, 1949). A Jersey bull was obtained from Morvi State (Kathiawar, India) in 1942 and another Jersey bull was imported from Australia in 1946.

By 1934, animals of Sindhi breeding were most numerous in the herd. At this time, cows of Sindhi, Jersey-Sindhi, Brown Swiss-Sindhi, Holstein-Sindhi and Guernsey-Sindhi were in the herd (Schneider, 1944). A policy of backcrossing the first crosses to the Sindhi breed was initiated about this time. The data used in the present study have come from observations on animals produced as a result of this policy.

Management of the herd

The management of the herd has been subject to all the ordinary variations resulting from the changes in economic conditions and from shifts in emphasis due to changes in personnel. During the period from 1934 to 1940, there was considerable emphasis on carrying out the breeding policy and the herd was managed so as to make most progress in that direction. In later years, this emphasis was apparently not maintained (Tandon, 1949). The author is most familiar with the management of the herd during the period from 1934 to 1940 and the following description pertains most exactly to this period.

The calves were weaned at birth (except a very few progeny of Sindhi dams). They were fed whole milk for the first four weeks and then gradually changed over to skim milk and grain. The amount of milk fed varied according to age and was generally adjusted to about one-tenth of body weight. The grains consisted of equal parts of wheat bran, ground barley, ground sorghum seed and linseed cake. A mineral mixture consisting of bone meal, lime and salt, with a trace of potassium iodide, was added to the grains. The roughage for calves varied according to season but often consisted of green Napier and Guinea grass and some hay made from grass-weeds. In general the quality of roughage was poor. The feeding of skimmilk was gradually discontinued after the calves were six months of age. The grain mixture was also changed at this time. Up to six months the calves were housed in an open shed with an attached paddock except during winter months when they were housed in a shed that had thatch siding and provided some shelter from winds.

At six months, the calves were transferred to other paddocks and kept in small groups of 15 to 20 animals in each paddock. These paddocks usually had a shed without walls where the animals could seek shelter from rain. The grain mixture fed to calves older than six months consisted of 50 percent wheat bran, 30 per cent mustard cake, 10 per cent linseed cake and 10 per cent crushed Bengal gram (chick pea). The mineral mixture was added at a 3 per cent rate. The feed allowances were computed according to the Morrison standard at monthly intervals. For some time, the concentrate mixture was fed individually to each animal, but the practice was discontinued because of the labour costs. The roughage consisted mainly of sorghum silage and was fed to the groups in open wooden troughs so that the animals could eat as much as they wanted. Rock salt and water were available to animals of all ages at all times. The heifers were bred whenever they came in heat after 14 months of age. Not many (except some Jersey crosses) bred that early. The usual age at calving was about two and a half years or later.

Type of records studied

Records of birth weight were kept from very early times. More regular records, particularly in respect to the height of animals, were started at the end of 1935 and were continued with fair regularity until about 1940. After this period the records were comparatively sketchy, probably because of shortage of help consequent to the war and change of personnel. The data used in this study cover the period from 1928 to 1948. The data consist of body weight and height at withers recorded in pounds and in inches and tenths respectively. Readings were taken soon after birth and then at monthly intervals up to six months of age. Observations were then taken at 12 months and at yearly intervals thereafter.

Numbers of records available

The records at birth are most numerous for both sexes. The smaller number of males recorded in Table 2 does not reflect differences in sex ratio, but rather the failure to keep records on all males born in the herd. Under the system of management, male calves (except those needed for breeding) were disposed of within three days after birth. That no observations were recorded on some of these animals accounts for the apparent discrepancy in sex ratio. This early disposal of many males also explains the extreme paucity of observations on males at ages later than at birth. Since records on growth of males were so few, only the birth weights and heights of males were studied.

The most numerous data were the weights of females. Table 2 shows that there was a progressive reduction in numbers as the age advanced. The reduction in the number of observations was most pronounced between birth and one month (195) and between 24 and 36 months (76). The first drop is partially explained by the fact that for the years 1928 to 1933, and also for 1948, figures for the weights at birth only were available. Culling soon after birth and again at first calving is probably responsible for most of these reductions. The weight records for females after 24 months are further complicated by the changes incidental to gestation and lactation. Because of these, only the weights up to 24

Table 2. Summary of the number of records available

	Age in months														
	0	1	2	3	4	5	6	12	24	36	48	60	72	84	96
Females															
Weight	480	285	285	278	272	265	258	258	252	176	132	77	33	13	4
Height	401	126	142	146	155	166	138	106	85	85	40	18	4	1	-
Males															
Weight	274	37	29	29	25	28	19	1	1	1	-	-	1	-	-
Height	239	22	17	15	11	17	11	2	1	1	-	-	-	-	-

months have been studied here. Consecutive weight records were available on 233 animals. The records for height were not only fewer but in most cases were not consecutive. Use has, therefore, been made of height records at birth, six months, 12, 24 and 36 months. The height records after this age (36 months) were too few to have much value.

Methods

Methods for analyzing non-orthogonal data

The nature of data in animal husbandry experiments frequently makes the use of simple statistical analysis inappropriate. The usual methods of analysis of variance and covariance are applicable where the distribution of subclass numbers is equal, nearly equal, or at least proportionate. Such data usually occur only in specially designed experiments where exact control can be maintained over numbers. In animal breeding work, involving large farm animals, economic considerations and natural limitations on the type of work prevent exact control over numbers of animals. Even in cases where special attempts are made to plan the work, biological limitations, such as chance variations in the sex ratio, often prevent obtaining equal or proportionate subclass numbers in all classifications. Methods of analysis must, therefore, be devised which will permit flexibility in the type of data and yet yield results that are unbiased and efficient.

Brandt (1932) presented a method of handling analysis of variance with disproportionate frequencies. Brown (1932-a) used this method and compared the results so obtained with those obtained by using ordinary methods. In another study (Brown 1932-b), she used these methods but drew no conclusions on the efficacy of the adjustments made. The method essentially consisted of making an adjustment which Brandt called "f" and defined as "the difference between the two-way border means". Yates (1933) presented a method of handling non-orthogonal data by the analysis of variance of two-way classifications. Later he (Yates, 1934) extended this to deal with data with multiple classification. He compared the method of fitting constants and the method of using weighted squares of means and stated that the latter method, although easier to follow, does not provide a test of interaction except in cases of 2 x S tables. Hendricks (1935) described the method of weighted squares of means in considerable detail.

Snedecor (1934) presented the method of expected subclass numbers for analyzing data with disproportionate subclass frequencies. The method consisted of making the subclass numbers proportional by obtaining the expected sub class numbers based on border totals, and using expected sub class sums based on border sums. Snedecor and Cox (1935) surveyed the various methods of dealing with data involving unequal or disproportionate sub class numbers. They also presented the method of expected sub class numbers in detail.

Hazel (1946) extended the method of fitting constants, presented by Yates (1934), to include independent variables having continuous distributions. Henderson (1948) presented this method in considerable detail and developed methods for obtaining estimates of components of variance from the sums of squares obtained by this procedure. Dunlop (1949) gave many numerical examples of computational procedures. This method has been used in this study. The method is based on least squares and essentially consists of fitting a constant for each variable studied. The fitted constant represents the deviation from the hypothetical mean (μ), attributable to the influence of that variable.

Grouping of the data

The data consisted of observations on purebred Sindhi animals and on animals that were various grades of crosses between Sindhi and the four European breeds of cattle mentioned. For convenience in handling, the data are divided in groups as follows:

Group a_0	Purebred animals of the Sindhi breed
Group a_1	Animals with 1/8 or less blood of European breeds of cattle
Group a_2	Animals with more than 1/8 but with 2/8 or less blood of European breeds of cattle
Group a_3	Animals with more than 2/8 but with 3/8 or less blood of European breeds of cattle
Group a_4	Animals with more than 3/8 but with 4/8 or less blood of European breeds of cattle
Group a_5	Animals with more than 4/8 but with 5/8 or less blood of European breeds of cattle
Group a_6	Animals with more than 5/8 but with 6/8 or less blood of European breeds of cattle
Group a_7	Animals with more than 6/8 but with 7/8 or less blood of European breeds of cattle
Group a_8	Animals with more than 7/8 blood of European breeds of cattle

In addition to the "a" constants mentioned above, the following symbols are used to designate the constants fitted for the breeds of the animals

b_0 Sindhi

b_1 Jersey

b_2 Brown Swiss

b_3 Holstein-Friesian

b_4 Guernsey

The joint "a" and "b" classification of the available data is shown in Table 3.

Table 3. Classes of data available

	b_0	b_1	b_2	b_3	b_4
a_0	Sindhi	-	-	-	-
a_1	-	1/8 Jersey	1/8 Brown Swiss	1/8 Holstein-Friesian	1/8 Guernsey
a_2	-	2/8 Jersey	2/8 Brown Swiss	2/8 Holstein-Friesian	2/8 Guernsey
a_3	-	3/8 Jersey	3/8 Brown Swiss	3/8 Holstein-Friesian	0
a_4	-	4/8 Jersey	4/8 Brown Swiss	4/8 Holstein-Friesian	4/8 Guernsey
a_5	-	5/8 Jersey	0	0	0
a_6	-	6/8 Jersey	6/8 Brown Swiss	6/8 Holstein-Friesian	0
a_7	-	0	0	0	0
a_8	-	0	0	0	0

The a_5 class is represented by animals of Jersey breeding only and the Guernsey breed is represented in a_1, a_2 and a_4 classes only. The actual numbers of animals in different subclasses varied at different ages for different sexes and were different for weight and for height. The pure bred Sindhi animals stand in a class by themselves. Since the Sindhi breed is native to India, it was thought desirable to use this as a standard of comparison. The data for Sindhi animals were analyzed independently so that the cross bred animals could be compared to the Sindhi. The results have been presented in the same set of tables for convenience and compactness.

The mathematical model

The first step in following the least squares method of analysis—or any analysis for that matter—is to develop a mathematical model which is considered to fit the situation under examination reasonably well. In this study the following model has been used:

$$x_{ijk} = \mu + a_i + b_j + e_{ijk}$$

where,

x_{ijk} - is the body weight (or height at withers) of the k th animal with the i th proportion of blood in the j th breed.

μ - is the hypothetical mean value of the population from which the sample under examination is assumed to be drawn. It is presumed to present a measure of the central population value free from the influences of other variables. If the number of observations were very large and if the number of breeds were large and if all theoretically possible proportions of blood were included in the data, the general mean of such data would approach μ. In this case because the proportions of blood and the breed are known factors, μ represents the mean of a population composed of equal numbers in each proportion of blood and from each breed included in the data.

a_i - is the effect of the i th proportion of blood of European breeds of cattle. It represents the deviation from μ attributable to the effect of the proportion of blood.

b_j - is the effect of the j th European breed of cattle. It is the deviation from μ attributable to the effect of that breed.

e_{ijk}- is the random variation from μ which cannot be attributed to a_i or b_j. In the analysis it is assumed that the e_{ijk} values are normally and independently distributed with mean = 0 and variance = σ^2

Based on this model, the following relationships are obtained:

$$\mu = x_{ijk} - a_i - b_j - e_{ijk}$$

$$a_i = x_{ijk} - \mu - b_j - e_{ijk}$$

$$b_j = x_{ijk} - \mu - a_i - e_{ijk}$$

$$e_{ijk} = x_{ijk} - \mu - a_i - b_j$$

The error sum of squares is correspondingly set out thus:

$$\sum_{ijk} e^2_{ijk} = \sum_{ijk} (x_{ijk} - \mu - a_i - b_j)^2$$

The least squares estimates of the various parameters are obtained in such a way that the sums of the squares of the deviations are minimum. This is accomplished by partially differentiating the error sum of squares with respect to each of the parameters and setting the differentials equal to zero. The following group of normal equations is obtained in this manner:

$$\mu : n_{..}\mu + \sum_{i=1} n_{i.}a_i + \sum_{j=1} n_{.j}b_j = x_{..}$$

$$a_i : n_{i.}(\mu + a_i) + \sum_{j=1} n_{ij}b_j = x_{i.}$$

$$b_j : n_{.j}(\mu + b_j) + \sum_{i=1} n_{ij}a_i = x_{.j}$$

These equations can be expanded and written out in detail to fit the present case as shown in Table 4. It will be observed that these equations are not independent. Thus the sum of a_i equations = the sum of b_j equations = the μ equation. As such, no solution is possible. If, however, the restriction that,

$$\sum a_i = \sum b_j = 0$$

is imposed, the dependence is eliminated and a solution can be obtained. The number of equations can be reduced by eliminating one equation and deducting the coefficient pertaining to it from the other coefficients in the information matrix. The estimates of the parameters are obtained by solving the remaining reduced equations. If the number of equations is at all large, the method of iteration is of considerable help in obtaining a solution. A shortcut method of solving the equations by absorbing parameters with μ and inverting the matrix has been described by Henderson (1949).

Table 4. Least squares equations

	μ	a_1	a_2	a_3	a_4	a_5	a_6	b_1	b_2	b_3	b_4		
μ	$n_{..}$	$n_{1.}$	$n_{2.}$	$n_{3.}$	$n_{4.}$	$n_{5.}$	$n_{6.}$	$n_{.1}$	$n_{.2}$	$n_{.3}$	$n_{.4}$	=	$x_{..}$
a_1	$n_{1.}$	$n_{1.}$						n_{11}	n_{12}	n_{13}	n_{14}	=	$x_{1.}$
a_2	$n_{2.}$		$n_{2.}$					n_{21}	n_{22}	n_{23}	n_{24}	=	$x_{2.}$
a_3	$n_{3.}$			$n_{3.}$				n_{31}	n_{32}	n_{33}	n_{34}	=	$x_{3.}$
a_4	$n_{4.}$				$n_{4.}$			n_{41}	n_{42}	n_{43}	n_{44}	=	$x_{4.}$
a_5	$n_{5.}$					$n_{5.}$		n_{51}	n_{52}	n_{53}	n_{54}	=	$x_{5.}$
a_6	$n_{6.}$						$n_{6.}$	n_{61}	n_{62}	n_{63}	n_{64}	=	$x_{6.}$
b_1	$n_{.1}$	n_{11}	n_{21}	n_{31}	n_{41}	n_{51}	n_{61}	$n_{.1}$				=	$x_{.1}$
b_2	$n_{.2}$	n_{12}	n_{22}	n_{32}	n_{42}	n_{52}	n_{62}		$n_{.2}$			=	$x_{.2}$
b_3	$n_{.3}$	n_{13}	n_{23}	n_{33}	n_{43}	n_{53}	n_{63}			$n_{.3}$		=	$x_{.3}$
b_4	$n_{.4}$	n_{14}	n_{24}	n_{34}	n_{44}	n_{54}	n_{64}				$n_{.4}$	=	$x_{.4}$

Analysis of variance

The analysis of variance is performed by the usual procedure described by Snedecor (1946). The various sums of squares are obtained as usual. The sums of squares for "a", "b" and "ab" are obtained by first computing the reduction due to fitting the constants, μ, a_i and b_j as follows:

Reduction due to fitting μ, $a_i\ b_j$ =

$$\left[x_{1.}(\mu + a_1) + x_{2.}(\mu + a_2) + x_{3.}(\mu + a_3) + x_{4.}(\mu + a_4) + x_{5.}(\mu + a_5) + x_{6.}(\mu + a_6)\right]$$

$$+ \left[x_{.1}(b_1) + x_{.2}(b_2) + x_{.3}(b_3) + x_{.4}(b_4)\right]$$

This is now called $R(\mu a_i b_j)$

Sum of squares for a_i =

$$R(\mu a_i b_j) - \left[(x_{.1})^2/n_{.1} + (x_{.2})^2/n_{.2} +++ (x_{.j})^2/n_{.j}\right]$$

Sum of squares for b_j =

$$R(\mu \bullet a_i b_j) - \left[(x_{1.})^2 n_{1.} + (x_{2.})^2/n_{2.} +++ (x_{i.})^2/n_{i.}\right]$$

Sum of squares for ab_{ij} =

$$\left[(x_{11})^2/n_{11} + (x_{12})^2/n_{12} ++++ (x_{ij})^2/n_{ij}\right] - \left[R(\mu a_i b_j)\right]$$

Having obtained the various sums of squares, the analysis of variance is carried out as usual.

In order to illustrate the procedure used in the analysis of variance, the numerical example of analysis of all data for birth weight of females is given. The sums, numbers and the symbols used are presented in Table 5.

Table 5. Weight at birth for all 480 females.
Numbers and sums in different classifications.

	b_0	b_1	b_2	b_3	b_4	Sum
a_0	$n_{00} = 115$ $x_{00} = 4885$	-	-	-	-	$n_{0.} = 115$ $x_{0.} = 4885$
a_1	-	$n_{11} = 80$ $x_{11} = 3308$	$n_{12} = 43$ $x_{12} = 2013$	$n_{13} = 43$ $x_{13} = 1998$	$n_{14} = 8$ $x_{14} = 345$	$n_{1.} = 174$ $x_{1.} = 7664$
a_2	-	$n_{21} = 76$ $x_{21} = 3511$	$n_{22} = 19$ $x_{22} = 1046$	$n_{23} = 7$ $x_{23} = 331$	$n_{24} = 5$ $x_{24} = 268$	$n_{2.} = 107$ $x_{2.} = 5156$
a_3	-	$n_{31} = 2$ $x_{31} = 58$	$n_{32} = 1$ $x_{32} = 50$	$n_{33} = 1$ $x_{33} = 65$	-	$n_{3.} = 4$ $x_{3.} = 173$
a_4	-	$n_{41} = 24$ $x_{41} = 885$	$n_{42} = 18$ $x_{42} = 825$	$n_{43} = 11$ $x_{43} = 499$	$n_{44} = 3$ $x_{44} = 152$	$n_{4.} = 56$ $x_{4.} = 2361$
a_5	-	$n_{51} = 11$ $x_{51} = 377$	-	-	-	$n_{5.} = 11$ $x_{5.} = 377$
a_6	-	$n_{61} = 7$ $x_{61} = 235$	$n_{62} = 2$ $x_{62} = 90$	$n_{63} = 4$ $x_{63} = 246$	-	$n_{6.} = 13$ $x_{6.} = 1187$
Sum	$n_{.0} = 115$ $x_{.0} = 4885$	$n_{.1} = 200$ $x_{.1} = 8374$	$n_{.2} = 83$ $x_{.2} = 4024$	$n_{.3} = 66$ $x_{.3} = 3139$	$n_{.4} = 16$ $x_{.4} = 765$	$n_{..} = 480$ $x_{..} = 21187$

It will be observed that the a_0b_0 subclass stands by itself. In the least squares procedure, no separate estimates of a_0 and b_0 constants can be obtained and, as mentioned earlier, this part of the data (pure-bred Sindhi animals) has been analyzed separately. The information matrix for cross bred females is presented in Table 6.

Since there are more "a" constants than "b" constants, some ease in computational procedure is obtained by absorbing the "a" constants together with "μ" and solving the remaining equations for the "b" constants. The absorption of "a" constants is achieved by multiplying the numbers in each cell of "b" classes by a coefficient which is the proportion of the number in that cell to the total number in the corresponding "a" class. The method of obtaining the coefficients is explained in Table 7.

The simultaneous equations now become:

$$
\begin{aligned}
b_1:\ 200 &- \left[80(.4598) + 76(.7103) + 2(.5000) + 24(.4286) + 11(1.000) + 7(.5385)\right] \\
&- \left[80(.2471) + 76(.1776) + 2(.2500) + 24(.3204) + 11(0\ \ ) + 7(.1538)\right] \\
&- \left[80(.2471) + 76(.0654) + 2(.2500) + 24(.1964) + 11(0\ \ ) + 7(.3077)\right] \\
&- \left[80(.0460) + 76(.0467) + 2(0\ \ ) + 24(.0536) + 11(0\ \ ) + 7(0\ \ )\right] \\
&= 8374 - \left[7664(.4598) + 5156(.7103) + 173(.5000) + 2361(.4286) \right. \\
&\qquad \left. + 377(1.0000) + 571(.5385)\right]
\end{aligned}
$$

$$
\begin{aligned}
b_2:\ \ &- \left[43(.4598) + 19(.7103) + 1(.5000) + 18(.4286) + 0(1.0000) + 2(.5385)\right] \\
83\ &- \left[43(.2471) + 19(.1776) + 1(.2500) + 18(.3204) + 0(0\ \ ) + 2(.1538)\right] \\
&- \left[43\ (.2471) + 19(.0654) + 1(.2500) + 18(.1964) + 0(0\ \ ) + 2(.3077)\right] \\
&- \left[43\ (.0460) + 19(.0467) + 1(0\ \ ) + 18(.0536) + 0(0\ \ ) + 2(0\ \ )\right] \\
&= 4024 - \left[7664(.2471) + 5156(.1776) + 173(.2500) + 2361(.3204) \right. \\
&\qquad \left. + 571(.1538)\right]
\end{aligned}
$$

Table 6. Information matrix for cross bred females; weight at birth

		a_1	a_2	a_3	a_4	a_5	a_6	b_1	b_2	b_3	b_4		
	365	174	107	4	56	11	13	200	83	66	16	=	16302
a_1		174						80	43	43	8	=	7664
a_2			107					76	19	7	5	=	5156
a_3				4				2	1	1	0	=	173
a_4					56			24	18	11	3	=	2361
a_5						11		11	0	0	0	=	377
a_6							13	7	2	4	0	=	571
b_1		80	76	2	24	11	7	200				=	8374
b_2		43	19	1	18	0	2		83			=	4024
b_3		43	7	1	11	0	4			66		=	3139
b_4		8	5	0	3	0	0				16	=	765

Table 7. Method of obtaining the proportionate coefficients for each cell in "a" classification for cross bred females; weight at birth

	b_1	b_2	b_3	b_4	Sum
a_1	80/174 = 0.4598	43/174 = 0.2471	43/174 = 0.2471	8/174 = 0.0460	174 1.0000
a_2	76/107 = 0.7103	19/107 = 0.1776	7/107 = 0.0654	5/107 = 0.0467	107 1.0000
a_3	2/4 = 0.5000	1/4 = 0.2500	1/4 = 0.2500	0	4 1.0000
a_4	24/56 = 0.4286	18/56 = 0.3214	11/56 = 0.1964	3/56 = 0.0536	56 1.0000
a_5	11/11 = 1.0000	0	0	0	11 1.0000
a_6	7/13 = 0.5385	2/13 = 0.1538	4/13 = 0.3077	0	13 1.0000

$$b_3: -\left[43(.4598) + 7(.7103) + 1(.5000) + 11(.4286) + 0(1.000) + 4(.5385)\right]$$

$$-\left[43(.2471) + 7(.1776) + 1(.2500) + 11(.3214) + 0(0\quad) + 4(.1538)\right]$$

66 $$-\left[43(.2471) + 7(.0654) + 1(.2500) + 11(.1964) + 0(0\quad) + 4(.3077)\right]$$

$$-\left[43(.0460) + 7(.0467) + 1(0\quad) + 11(.0536) + 0(0\quad) + 4(0\quad)\right]$$

$$= 3139 - \left[7664(.2471) + 5156(.0654) + 173(.2500) + 2361(.1964) + 377(0\quad) + 571(.3077)\right]$$

$$b_4: -\left[8(.4598) + 5(.7103) + 0(.5000) + 3(.4286) + 0(1.0000) + 0(.5385)\right]$$

$$-\left[8(.2471) + 5(.1776) + 0(.2500) + 3(.3214) + 0(0\quad) + 0(.1538)\right]$$

$$-\left[8(.2471 + 5(.0654) + 0(.2500) + 3(.1964) + 0(0\quad) + 0(.3077)\right]$$

16 $$-\left[8(.0460) + 5(.0467) + 0(0\quad) + 3(.0536) + 0(0\quad) + 0(0\quad)\right]$$

$$= 765 - \left[7664(.0460) + 5156(.0467) + 173(0\quad) + 2361(.0536) + 377(0\quad) + 571(0\quad)\right]$$

When solved, the following equations are obtained:

$$b_1: \; + 83.1821b_1 - 42.5566b_2 - 32.1102b_3 - 8.5153b_4 = -594.7024$$

$$b_2: \; - 42.5566b_1 + 62.6563b_2 - 16.2707b_3 - 3.8291b_4 = +324.4826$$

$$b_3: \; - 32.1102b_1 - 16.2705b_2 + 51.2741b_3 - 2.8934b_4 = +225.0044$$

$$b_4: \; - 8.5153b_1 - 3.8291b_2 - 2.8934b_3 + 15.2378b_4 = + 45.2154$$

It will be observed that the figures on the left of the equality sign are entirely symmetrical and add up to zero, both vertically and horizontally. The figures on the right of the sign also add up to zero. This provides a convenient check on the computations. It also means that the equations are not independent and cannot be solved in this form.

The independence may be easily obtained by deducting the set of coefficients immediately to the left of the equal sign from the other coefficients of the same equation. After this, the equation for b_4 becomes redundant and may be deleted from the set entirely. The remaining three equations now are:

b_1: $+ 91.6974b_1 - 34.0413b_2 - 23.5949b_3 = - 594.7024$

b_2: $- 38.7275b_1 + 66.4854b_2 - 12.4414b_3 = + 324.4826$

b_3: $- 29.2168b_1 - 13.3771b_2 + 54.1675b_3 = + 225.0044$

The iterative solution of these equations can now be easily obtained by transposing the quantities on the right of the equal sign to the left and making the equations equal to zero. The iterative solution of these equations is presented in Table 8.

Table 8. Iterative solution of equations to obtain estimates of "b" constants for cross bred females; weight at birth

	b_1	b_2	b_3	
	-5.1622	+ 2.2329	+ 1.9209	
Successive estimates of "b" constants	-5.1622	+ 2.2330	+ 1.9209	
	-5.1622	+ 2.2330	+ 1.9209	
	-5.1619	+ 2.2333	+ 1.9212	
	-5.161	+ 2.234	+ 1.922	
	-5.160	+ 2.235	+ 1.925	
	-5.15	+ 2.24	+ 1.93	
	-5.10	+ 2.28	+ 1.97	
	-5.0	+ 2.4	+ 2.1	
	-5.2	+ 2.1	+ 1.9	
	-6.4	+ 1.3	+ 1.3	
		0	0	
b_1	+91.6974	-34.0413	-23.5949	+ 594.7024 = 0
b_2	-38.7275	+66.4854	-12.4414	- 324.4826 = 0
b_3	-29.2168	-13.3771	+54.1675	- 225.0044 = 0

Having obtained the estimates of b_1, b_2 and b_3, the value of b_4 can be obtained by difference because of the assumption, $b_j = 0$. The estimated values are:

$$b_1 = -5.1622$$

$$b_2 = +2.2330$$

$$b_3 = +1.9209$$

$$b_4 = +1.0083$$

The next step is to obtain estimates of the "a" constants as follows:

$$\mu + a_1 = \{7664 - [80(-5.1622) + 43.(2.2330) + 43(1.9209) + 8(1.0083)]\}/174$$
$$= 7890.2919/174 \qquad = 45.3465$$

$$\mu + a_2 = \{5156 - [76(-5.1622) + 19(2.2330) + 7(1.9209) + 5(1.0083)]\}/107\}$$
$$= 5087.4124/107 \qquad = 51.2842$$

$$\mu + a_3 = \{173 - [2(-5.1622) + 1(2.2330) + 1(1.9209) + 0(1.0083)]\}/4\}$$
$$= 179.1705/4 = \qquad = 44.7926$$

$$\mu + a_4 = \{2361 - [24(-5.1622) + 18(2.2330) + 11(1.9209) - 3(1.0083)]\}\ldots/56 = 43.2240$$

$$\mu + a_5 = \{377 - [11(-5.1622) + 0(2.2330) + 0(1.9209) + 0(1.0083)]\}/11$$
$$= 433.7842/11 \qquad = 39.4349$$

$$\mu + a_6 = \{571 - [7(-5.1622) + 2(2.2330) + 4(1.9209) + 0(1.0083)]\}/13$$
$$= 594.9858/13 \qquad = 45.7681$$

Since $\Sigma a_i = 0$, $\mu = \sum_i (\mu + a_i)/n_i = 269.8503/6 = 44.9750$

The estimates of the "a" constants are now obtained by deducting "μ" from the "$\mu + a$" quantities.

$a_1 = 45.3465 - 44.9750 = + 0.3715$

$a_2 = 51.2842 - 44.9750 = + 6.3092$

$a_3 = 44.7926 - 44.9750 = - 0.1824$

$a_4 = 43.2240 - 44.9750 = - 1.7510$

$a_5 = 39.4349 - 44.9750 = - 5.5401$

$a_6 = 45.7681 - 44.9750 = + 0.7931$

For the analysis of varience, the sums of squares are first obtained by the usual methods. These are:

Total		41567.6
Between Sindhi and Crosses		417.3
Remainder		41150.3
Within Sindhi		7752.7
Within Crosses		33397.6
Subclasses		8422.4
Proportions (a)	2948.2	
Breeds (b)	3483.6	
Interactions (ab)	990.6	
Residual		24975.2

These sums of squares for "a", "b" and "ab" are not appropriate because the ordinary computational procedure does not recognize the unequal subclass distribution. The least squares estimates of sums of squares for proportions, breeds and interaction are obtained by first computing the reduction due to fitting the constants, μ, a_i and b_j and then deducting the sums of squares due to breeds and due to proportions of blood from it. This gives the estimates for sums of squares due to proportions of blood and breeds respectively. The interaction sum of squares is obtained by deducting the reduction due to fitting constants from the subclass sum of squares. These operations are carried out thus:

Reduction due to fitting μ, a_i, b_j =

$$[7664(45.3465) + 5156(51.2842) + 173(44.7926) + 2361(43.2240) + 377(39.4349) + 571(45.7681)]$$

$$+ [8374(-5.1622) + 4024(2.2330) + 3139(1.9209) + 765(1.0083)]$$

$$- \text{correction term}$$

$$= [762758.4374] + [-27441.6162] - [728096.4] = 7220.4$$

"a" sum of squares = 7220.4 - 3483.6 = 3736.8

"b" sum of squares = 7220.4 - 2948.2 = 4272.2

8009.0

"ab" sum of squares = 8422.4 - 7220.4 = 1202.0

It will be observed that these sums of squares do not agree with those obtained by the usual computational procedure (p. 34). This happens because the least squares estimates of these sums of squares are obtained as if the numbers in the subclasses were equal. Analysis of variance follows immediately and is presented in Table 9.

Analysis of covariance

Analysis of covariance between weight at one age and the corresponding weight at another age was performed on the data for females only. Consecutive weights for 233 females were available from birth to 24 months. The main purpose of the analysis was to determine the amount of precision obtained in forecasting the weight at a later age from the statistical control over the weight at an earlier age. Weight at the later age was the dependent variable and the weight at an earlier age was the independent variable. The usual methods of covariance and

Table 9. Analysis of variance for weight at birth of all 480 females

Source	Degrees of freedom	Sum of squares	Mean square	"F" ratio
Total	479	41567.6	-	
Between Sindhi and Crosses	1	417.3	417.30	417.30/86.08=4.85*
Remainder	478	41150.3	86.08	
Within Sindhi	114	7752.7	68.00	
Within Crosses	364	33397.6	91.75	
Proportions (a)	5	3736.8	747.36	747.36/72.18 = 10.35**
Breeds (b)	3	4272.2	1424.07	1424.07/72.18 = 19.73**
Interaction (ab)	10	1202.0	120.20	120.20/72.18 = 1.66
Residual (error)	346	24975.2	72.18	

*Denotes a probability level of .05 or lower.

**Denotes a probability level of .01 or lower.

regression analysis were used except that least squares estimates of the covariances for "a", "b" and "ab" were obtained by extension of methods described before.

Reduction in products due to fitting $\mu\, a_i b_j$, (abbreviated to $R_p\ (\mu a_i b_j)$ hereafter =

$$\tfrac{1}{2}\left\{\sum_{i=1}^{i}\left[x_{i.}\ (\mu' + a_i') + x_{i.}'\ (\mu + a_i)\right] + \sum_{j=1}^{j}\left[x_{.j}(b_j') + x'_{.j}(b)\right]\right\}$$

$$- \left[(x_{..})\ (x'_{..})/n_{..}\right]$$

where

μ and μ'

a_i and a_i'

b_j and b_j'

$x_{.j}$ and $x_{.j}'$

$x_{..}$ and $x'_{..}$

represent the constants and observations at two different ages.

The expression is general and can be expanded for the present case.

$$R_p\ (\mu\, a_i b_j) =$$

$$\begin{aligned}\tfrac{1}{2}\Big\{\big[&x_{1.}(\mu' + a_1') + x_{1.}'\ (\mu + a_1) + x_{2.}(\mu' + a') + x_{2.}'(\mu + a_2)\\ &+ x_{4.}\ (\mu' + a_4') + x_{4.}'\ (\mu + a_4) + x_{5.}\ (\mu' + a_5') + x_{5.}'\ (\mu + a_5)\\ &+ x_{6.}\ (\mu' + a_6') + x_{6.}'\ (\mu + a_6)\big] + \big[x_{.1}\ (b_1') + x'_{.1}\ (b_1)\\ &+ x_{.2}\ (b_2') + x'_{.2}(b_2) + x_{.3}(b_3') + x'_{.3}(b_3) + x_{.4}(b_4')\\ &+ x'_{.4}(b_4)\big]\Big\} - \left[(x_{..})(x'_{..})/n_{..}\right]\end{aligned}$$

Products for a_i =

$$R_p(\mu a_i b_j) - \{[(x._1)(x'_1)/n._1 + (x_{.2})(x'_2)/n_2 +++ (x_j)(x'_j)/n_{.j}] - [(x..)(x'..)/n..]\}$$

Products for b_j =

$$R_p(\mu a_i b_j) - \{[(x_{1.})(x'_{1.})/n_{1.} + (x_{2.})(x'_{2.})/n_{1.} +++ (x_{i.})(x'_{i.})/n_{i.}] - [(x..)(x'..)/n..]\}$$

Products for ab_{ij} =

$$\{[(x_{11})(x'_{11})/n_{11} + (x_{12})(x'_{12})/n_{12} ++++ (x_{ij})(x'_{ij})/n_{ij}] - [(x..)(x'..)/n..]\} - R_p\ (\mu a_i b_j)$$

The following numerical example on covariance between weight at birth and weight at one month, for records that were consecutive up to 24 months of age (n = 233), is given to illustrate the procedure.

The values of various cross products were:

Total	14937.3
Between Sindhi and Crosses	291.3
Remainder	14646.0
Within Sindhi	3546.6
Within Crosses	11099.4
Subclasses	4826.1
Proportions of blood (a)	2593.5
Breeds (b)	2076.7
Interactions (ab)	155.9
Residual	6273.3

Values of the constants, border sums and numbers are given in Table 10.

Table 10. Values of constants, border sums and numbers. Weight at birth and at one month for 175 cross bred females

		Age			
		Birth		One month	
μ		43.8328		60.8592	
a_1		+ 5.3224		+ 6.5331	
a_2		- 9.3395		+ 6.6012	
a_4		- 1.6347		- 0.0571	
a_5		- 3.7412		- 0.4222	
a_6		- 9.2858		-12.6548	
b_1		- 5.6630		- 2.0084	
b_2		+ 3.8009		+ 4.2076	
b_3		+ 0.1740		+ 5.1380	
b_4		+ 1.6881		- 7.3372	
$n_{1.}$,	$x_{1.}$	86	4099	86	5901
$n_{2.}$,	$x_{2.}$	65	3266	65	4354
$n_{4.}$,	$x_{4.}$	13	526	13	791
$n_{5.}$,	$x_{5.}$	7	241	7	409
$n_{6.}$,	$x_{6.}$	4	125	4	191
$n_{.1}$,	$x_{.1}$	100	4370	100	6389
$n_{.2}$,	$x_{.2}$	41	2185	41	2891
$n_{.3}$,	$x_{.3}$	27	1341	27	1952
$n_{.4}$,	$x_{.4}$	7	361	7	414

From these the reduction in products due to fitting constants, μ, a_i and b_j is computed thus:

$$R_p(\mu a_i b_j) =$$

$$\tfrac{1}{2}\{[4099\ (67.3923) + 5901\ (49.1552) + 3266\ (67.4604)$$

$$+ 4354\ (53.1723) + 526\ (60.8021) + 791\ (42.1981)$$

$$+ 241\ (60.4370) + 409\ (40.0916) + 125\ (48.2044)$$

$$+ 191\ (34.5470)\Big] + \Big[6389\ (-5.6630) + 4370\ (-2.0084)$$

$$2891\ (3.8009) + 2185\ (4.2076) + 1952\ (0.1740)$$

$$1341\ (5.1380) + 414\ (1.6881) + 361\ (-7.3372)\Big]\Big\}$$

$$-\ 549491.6$$

$$= \tfrac{1}{2}\Big\{\Big[1127091.1\Big] + \Big[-19495.8\Big]\Big\} - \Big[549491.6\Big]\Big\}$$

$$= 553797.7 - 549491.6 = 4306.1$$

"a" products, 4306.1 - 2076.7 = 2229.4

"b" products, 4306.1 - 2593.5 = 1712.6

"ab" products, 4826.1 - 4306.1 = 520.0

As in the analysis of variance, the "a", "b" and "ab" products estimated by the least squares procedure do not agree with the estimates of these by the usual method (p. 38). The reason for the difference is that the figures obtained by the usual method as computed on the assumption that there is no unequal subclass distribution in the data. The estimates so obtained are therefore inefficient. The least squares procedure takes the disproportionality of the distribution into account.

The analysis of covariance follows and is presented in Table 11.

Results

Males

Because the data on males were extremely limited, no growth study could be made. The data for height and for weight at birth were most numerous. Analyses of variance have been carried out for these. A total of 239 records for height and 274 records for weight were available. The distribution of these is shown in Tables 12 and 13.

Table 11. Analysis of covariance for weights of females at birth and at one month. The dependent variable is weight at one month (x').

		Degrees of freedom	Squares and products S_x^2	$S_{xx'}$	$S_{x'}^2$	Errors of estimate Degrees of freedom	Sum of squares	Mean square
Total		232	21871.9	14937.3	29289.2	231	19087.8	
Between Sindhi and Crosses		1	505.7	291.7	167.8			
Remainder		231	21366.2	14646.0	29121.4	230	19081.9	82.96
Difference						1	5.9	5.90
Within Sindhi		57	4430.1	3546.6	6534.1			
Within Crosses		174	16936.1	11099.4	22587.3			
Proportions	(a)	4	3090.3	2229.4	2097.6			
Breeds	(b)	3	2764.9	1712.6	2046.6			
Interactions	(ab)	7	837.4	520.0	600.1			
Residual	(error)	160	9962.7	6273.3	17262.8	159	13312.6	83.73
Proportions plus Residual		164	13053.0	8502.7	19360.4	163	13821.8	
Difference for "a"						4	509.2	127.32
Breeds plus Residual		163	12727.6	7985.9	19309.4	162	14298.8	
Difference for "b"						3	986.2	328.73
Interactions plus Residual		167	10800.1	6793.3	17862.9	166	13589.9	
Difference for "ab"						7	277.3	39.61

Table 12. Distribution of data for males; height at birth

	b_0	b_1	b_2	b_3	b_4	Sum
a_0	66	-	-	-	-	66
a_1	-	56	31	14	4	105
a_2	-	32	8	2	0	42
a_3	-	4	0	0	0	4
a_4	-	6	0	0	0	6
a_5	-	11	0	0	0	11
a_6	-	5	0	0	0	5
Sum	66	114	39	16	4	239

Table 13. Distribution of data for males; weight at birth

	b_0	b_1	b_2	b_3	b_4	Sum
a_0	72	-	-	-	-	72
a_1	-	56	31	14	4	105
a_2	-	33	11	2	0	46
a_3	-	4	0	0	0	4
a_4	-	13	11	3	1	28
a_5	-	11	0	0	0	11
a_6	-	5	0	3	0	8
Sum	72	122	53	22	5	274

The various constants computed from the data are presented in Table 14.

Table 14. Values of various constants for weight and for height at birth for males

	Height (inches)	Weight (pounds)
Sindhi (mean)	25.21$\pm$0.14	44.5$\pm$0.9
Crosses		
μ	24.49	44.8
a_1	+ 0.70	+ 9.54
a_2	+ 0.84	+ 5.94
a_3	+ 0.45	+ 0.63
a_4	+ 0.05	- 0.79
a_5	- 1.35	- 8.91
a_6	- 0.69	+ 2.59
b_1	- 0.26	- 2.45
b_2	+ 0.80	+ 6.21
b_3	- 0.44	- 0.67
b_4	- 0.10	- 3.09

The mean squares in the analysis of variance are given in Table 15.

Table 15. Mean squares in the analyses of variance for height and weight at birth of males

		Height		Weight	
		Degrees of freedom	Mean square	Degrees of freedom	Mean square
Between Sindhi and Crosses		1	2.21	1	110.4
Remainder		237	2.43	272	104.0
Within Sindhi		65	1.38	71	54.2
Within Crosses		172	-	201	-
Proportions	(a)	5	10.37**	5	465.1**
Breeds	(b)	3	11.24**	3	890.8**
Interactions	(ab)	2	2.62	6	157.1
Residual	(error)	162	2.46	187	92.6

**Denotes a probability level of 0.01 or lower.

Females

As already mentioned, the data on females were more numerous than the data on males. There were sizable decreases in the number of observations between birth and one month and between 24 months and 36 months. For these reasons and because the weights after 24 months may have been influenced by pregnancy or lactation, the data for weight were analyzed up to 24 months only. The data for height were studied up to 36 months, there being no drop in the number of observations on height between 24 and 36 months (Table 2).

Height. The observations on individual animals were not taken with consistent regularity. Data were missing at one or more ages for many of the animals. Consecutive records were available for very few animals and because of this the covariance between observations at different ages was not studied. The distribution of records at different ages is shown in Table 16.

Table 16. Distribution of data on height of females at various ages

Breed and proportion		Age (months)									
		0	1	2	3	4	5	6	12	24	36
b_0	a_0	103	37	32	34	39	36	28	21	16	18
b_1	a_1	81	22	18	22	24	32	24	14	7	10
	a_2	70	20	35	33	34	36	37	32	31	25
	a_3	2	0	0	0	0	0	0	0	0	0
	a_4	7	1	4	2	6	6	3	4	2	1
	a_5	11	2	1	4	5	3	4	0	0	0
	a_6	7	0	2	0	1	1	0	0	0	0
b_2	a_1	43	15	19	19	15	18	13	12	8	10
	a_2	18	8	8	8	8	10	11	9	7	6
	a_3	1	0	0	0	0	0	0	0	0	0
	a_4	1	0	0	1	1	0	1	0	0	0
b_3	a_1	42	17	19	18	17	20	12	10	9	11
	a_2	4	0	1	1	1	1	2	3	3	3
	a_4	0	1	1	1	0	0	0	0	0	0
	a_6	0	0	0	0	0	0	0	0	1	0
b_4	a_1	8	2	2	2	2	4	2	0	0	0
	a_2	3	0	0	1	2	2	1	1	1	1
Sum		401	126	142	146	155	166	138	106	85	85

The means of observations on Sindhi and on crosses are stated in Table 17, and a graphical representation is shown in Figure 1. It seems obvious from Figure 1 that the Sindhi and the cross breds grow in height at almost the same rate.

Table 17. Mean heights and standard errors of Sindhi and cross bred females from birth to three years

Age (months)	Sindhi Mean (inches)	Sindhi Number of observations	Crosses Mean (inches)	Crosses Number of observations
0	24.85±0.15	103	24.94±0.10	298
1	26.60±0.25	37	26.81±0.18	89
2	28.01±0.27	32	28.12±0.15	110
3	28.50±0.35	34	29.40±0.19	112
4	30.09±0.32	39	30.43±0.21	116
5	31.54±0.32	36	31.90±0.18	130
6	32.74±0.24	28	33.32±0.18	110
12	37.23±0.52	21	37.68±0.26	85
24	43.14±0.31	16	42.87±0.18	69
36	43.94±0.37	18	44.41±0.21	67

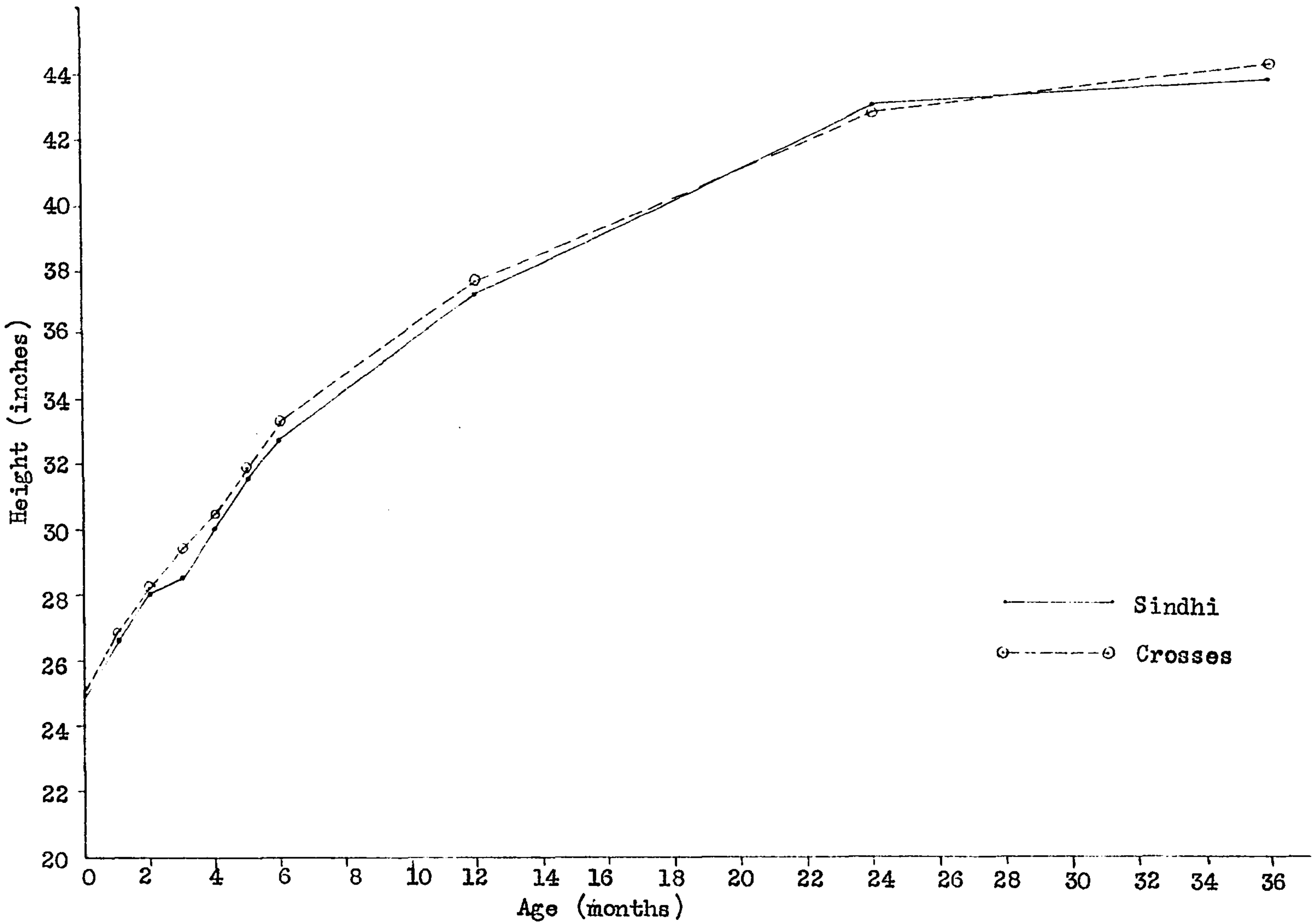

Figure 1. Growth of Sindhi and cross bred females in height at withers.

Analyses of variance were carried out on observations at birth, six months, 12 months, 24 months and 36 months. The various constants for the crosses are presented in Table 18 and the analyses of variance are given in Table 19.

Table 18. Values of various constants for height (in inches) of cross bred females

Constant	Age (months)				
	0	6	12	24	36
μ	24.16	33.44	37.73	42.99	44.71
a_1	+ 0.76	- 0.15	- 0.89	- 0.14	- 0.77
a_2	+ 1.50	+ 0.37	+ 0.22	+ 0.30	- 0.32
a_3	- 0.55	-	-	-	-
a_4	- 0.49	+ -.22	+ 0.67	+ 0.51	+ 1.09
a_5	- 0.75	- 0.44	-	-	-
a_6	- 0.47	-	-	- 0.67	-
b_1	- 0.33	- 0.32	- 0.02	- 0.60	- 0.50
b_2	+ 0.31	- 0.44	+ 0.07	- 0.19	+ 0.71
b_3	+ 0.42	+ 0.75	+ 1.20	+ 0.68	+ 1.67
b_4	- 0.40	- 0.01	- 1.25	+ 0.11	- 1.88

Table 19. Mean squares in the analyses of variance for height of females

Source of variation	Age (months)									
	0		6		12		24		36	
	Degrees of freedom	Mean square	Degrees of freedom	Mean square	Degrees of freedom	Mean square	Degrees of freedom	Mean square	Degrees of freedom	Mean square
Between Sindhi and crosses	1	0.67	1	7.59	1	3.46	1	0.99	1	3.18
Remainder	399	2.85	136	4.08	104	5.86	83	2.15	83	2.80
Within Sindhi	102	2.37	27	5.36	20	5.62	15	1.50	17	2.48
Within crosses	297	-	109	-	84	-	68	-	66	-
Proportions (a)	5	20.43**	3	2.76	2	8.88	3	2.01	2	3.04
Breeds (b)	3	10.00**	3	4.78	3	3.33	3	4.10	3	15.83**
Interactions (ab)	5	5.94	4	2.88	2	4.24	2	2.40	2	0
Residual (error)	284	2.57	99	3.84	77	6.01	60	2.23	59	2.40

**Denotes a probability level of 0.01 or lower.

Weight. The data for the weight of females were most numerous and most complete of all observations. The means of observations on Sindhi and crosses are presented in Table 20 and shown in Figure 2. Obviously the cross breds were a bit heavier but analyses of variance and covariance were needed for testing the statistical significance of the difference.

Table 20. Mean weights and standard errors of Sindhi and cross bred females from birth to two years

Age (months)	Sindhi		Crosses	
	Mean	Number of observations	Mean	Number of observations
0	42.5±0.8	115	44.7±0.6	365
1	64.3±1.3	67	65.8±0.8	218
2	83.0±1.8	68	87.4±1.2	217
3	104.9±3.0	66	113.3±1.5	212
4	130.7±3.4	64	139.6±2.0	208
5	152.8±4.4	61	166.2±2.4	204
6	175.6±4.8	63	194.5±2.5	195
12	312.8±12.0	59	332.8±6.0	199
24	446.6±12.6	58	500.5±6.8	194

These analyses were carried out on records of 233 animals that had complete records from birth to 24 months of age. The various constants for the crosses are presented in Table 21 and the mean squares in the analyses of variance are given in Table 22.

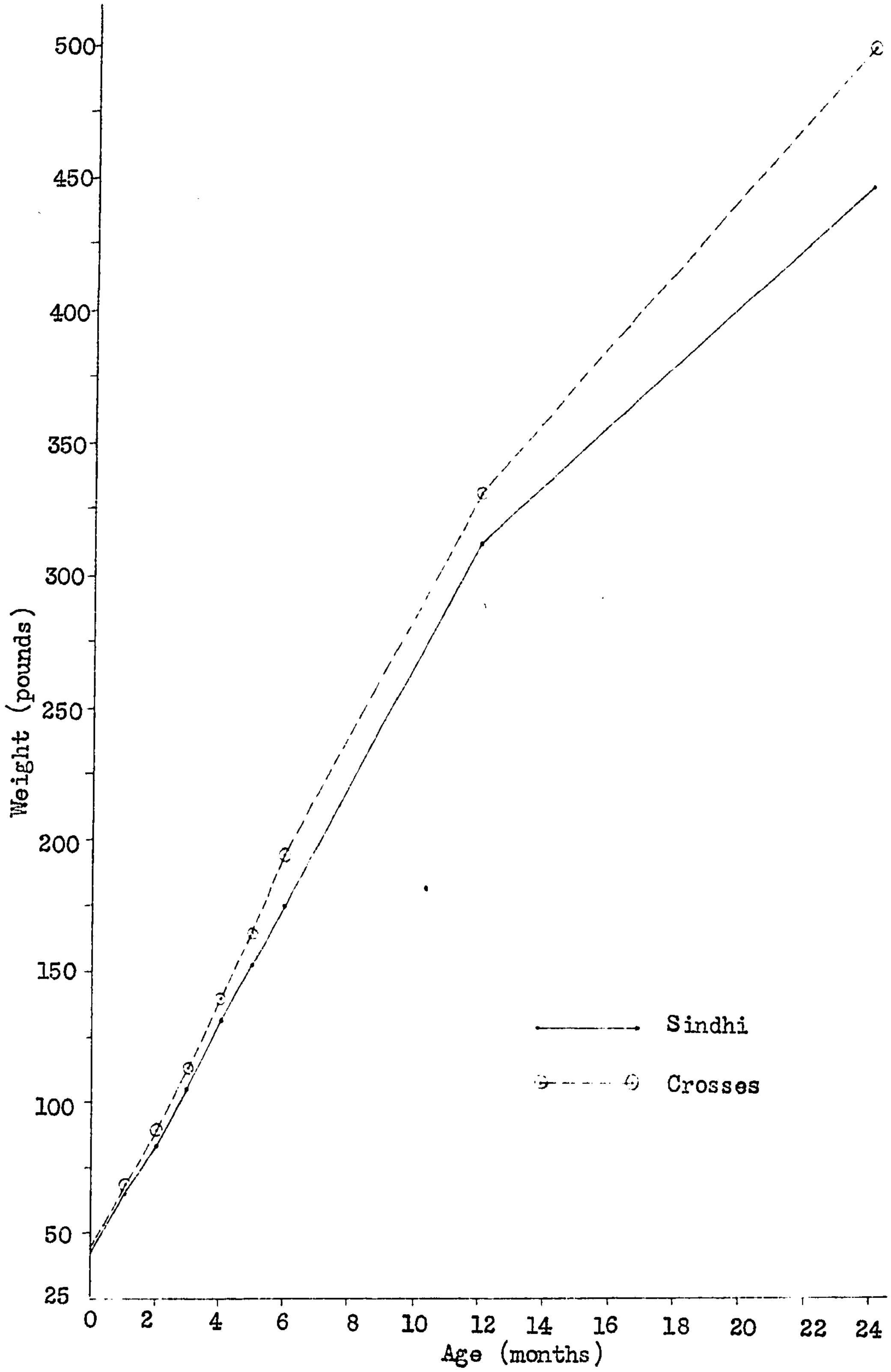

Figure 2. Growth of Sindhi and cross bred females in weight.

Table 21. Values of various constants for weight (in pounds) of cross bred females

	Age (months)								
	0	1	2	3	4	5	6	12	24
μ	43.83	60.86	87.16	110.00	140.51	165.58	195.73	336.92	510.20
a_1	+ 5.32	+ 6.53	+ 1.48	- 0.85	- 8.14	- 7.94	-14.03	-34.23	-28.09
a_2	+ 9.34	+ 6.60	+ 7.09	+ 7.70	+ 6.67	+10.52	+ 6.38	+ 9.20	+30.21
a_4	- 1.63	- 0.06	+ 1.44	+ 6.11	+ 8.43	+13.33	+13.23	+45.73	+55.60
a_5	- 3.74	- 0.42	+ 3.60	+10.29	+ 6.00	- 1.21	+ 3.86	+22.21	+ 6.65
a_6	- 9.29	-12.65	-13.61	-23.25	-12.96	-14.70	- 9.44	-42.91	-64.37
b_1	- 5.66	- 2.01	- 5.90	- 2.85	- 5.23	- 3.23	- 3.31	- 6.85	-24.43
b_2	+ 3.80	+ 4.21	+ 1.52	+ 3.58	+ 6.71	+ 8.16	+ 9.76	+ 5.50	+ 0.06
b_3	+ 0.17	+ 5.14	+11.00	+14.36	+15.75	+16.09	+16.01	+52.36	+28.62
b_4	+ 1.69	- 7.34	- 6.62	-15.09	-17.23	-21.02	-22.46	-51.01	- 4.14

Table 22. Mean squares in the analyses of variance for weight of females

Source of variation	Degrees of freedom	Age (months)						
		0	1	2	3	4	5	
Between Sindhi and crosses	1	505.7*	167.8	1402.0*	2523.8*	3088.8	10013.7**	13
Remainder	231	92.5	126.1	273.0	550.8	831.8	1191.7	1
Within Sindhi	57	77.7	114.6	233.8	636.3	801.3	1230.3	1
Within crosses	174							
Proportions (a)	4	772.6**	524.4**	597.7	1440.0*	2587.1**	3706.6**	4
Breeds (b)	3	921.6**	682.2**	2029.4**	2572.3**	4296.5**	4132.7*	4
Interactions (ab)	7	119.6	85.7	144.6	557.3	1290.3	1276.5	1
Residual (error)	160	62.3	107.9	254.0	469.7	742.8	1083.1	1

*Denotes a probability level of 0.05 or lower.

**Denotes a probability level of 0.01 or lower.

'es in the analyses of variance for weight of females

es	Age (months)								
om	0	1	2	3	4	5	6	12	24
	505.7*	167.8	1402.0*	2523.8*	3088.8	10013.7**	13252.7**	15037.0	129851.0**
	92.5	126.1	273.0	550.8	831.8	1191.7	1570.6	6441.1	8042.6
	77.7	114.6	233.8	636.3	801.3	1230.3	1432.4	8516.0	9151.3
	772.6**	524.4**	597.7	1440.0*	2587.1**	3706.6**	4693.2*	29599.2**	36410.2**
	921.6**	682.2**	2029.4**	2572.3**	4296.5**	4132.7*	4592.8*	29614.0**	9758.7
	119.6	85.7	144.6	557.3	1290.3	1276.5	1385.3	4563.3	11161.3
	62.3	107.9	254.0	469.7	742.8	1083.1	1529.0	5002.9	6780.7

:vel of 0.05 or lower.

:vel of 0.01 or lower.

The analyses of covariance between weight at an earlier age and the weight at a later age were carried out for the data on 233 females for which consecutive records were available. In these analyses the weights at later ages were considered to be the dependent variables and the weights at earlier ages were considered the independent variables. The usefulness of knowledge of weight at an earlier age in predicting the weight at a later age is indicated by the size of the error mean squares in the analysis of covariance. These are presented in Table 23.

Table 23. Analyses of covariance between weights of females at different ages (n =

Age (months)	Source	Degrees of freedom	Errors of estimate mean squares Ages (months) 1	2	3	4	5
0	Between Sindhi and crosses	1	5.9	1304.7*	2385.3*	1490.4	6631.0*
	Remainder	230	83.0	269.2	548.3	785.3	1139.5
	Proportions (a)	4	127.3	226.8	1105.0*	2386.9*	3239.0*
	Breeds (b)	3	328.7**	1266.8**	2050.4**	3197.5**	3095.8*
	Interactions (ab)	7	39.6	132.5	511.0	1160.1	1276.5
	Residual (error)	159	83.7	231.3	445.0	733.0	1066.9
1	Between Sindhi and crosses	1		493.5*	1326.4	1945.1	7853.2**
	Remainder	230		100.7	411.2	738.3	1102.3
	Proportions (a)	4		478.9**	1299.6**	2857.2**	3881.1**
	Breeds (b)	3		469.5**	676.8	1775.4	1732.4
	Interactions (ab)	7		184.1*	550.9	1076.8	1135.6
	Residual (error)	159		69.2	323.9	667.7	1016.2
2	Between Sindhi and crosses	1			66.9	2459.7*	2921.5
	Remainder	230			208.8	477.9	793.4
	Proportions (a)	4			285.3	1248.7*	1993.4*
	Breeds (b)	3			291.3	531.6	769.9
	Interactions (ab)	7			261.9	878.0	698.0
	Residual (error)	159			157.1	427.7	754.1
3	Between Sindhi and crosses	1				18.8	2246.3
	Remainder	230				260.5	599.5
	Proportions (a)	4				661.4*	1349.5
	Breeds (b)	3				2120.4**	267.7
	Interactions (ab)	7				1775.7**	410.2
	Residual (error)	159				234.8	645.4

*Denotes a probability level of 0.05 or lower.

**Denotes a probability level of 0.01 or lower.

a.

riance between weights of females at different ages (n = 233)

	Degrees of freedom	Errors of estimate mean squares							
		Ages (months)							
		1	2	3	4	5	6	12	24
ses	1	5.9	1304.7*	2385.3*	1490.4	6631.0*	9193.7*	13415.9	104515.0**
	230	83.0	269.2	548.3	785.3	1139.5	1517.2	6463.7	77874.3
	4	127.3	226.8	1105.0*	2386.9*	3239.0*	4800.5*	29620.9**	34589.7**
	3	328.7**	1266.8**	2050.4**	3197.5**	3095.8*	3318.5	29743.0**	4449.6
	7	39.6	132.5	511.0	1160.1	1276.5	1215.6	4621.9	10562.5
	159	83.7	231.3	445.0	733.0	1066.9	1494.3	5031.0	6735.5
ses	1		493.5*	1326.4	1945.1	7853.2**	10919.2**	14904.7**	102771.5**
	230		100.7	411.2	738.3	1102.3	1497.0	6461.4	8074.0
	4		478.9**	1299.6**	2857.2**	3881.1**	5474.2**	29745.1**	36280.0**
	3		469.5**	676.8	1775.4	1732.4	1744.8	25998.6**	9917.0
	7		184.1*	550.9	1076.8	1135.6	1327.4	4486.8	11285.0
	159		69.2	323.9	667.7	1016.2	1444.9	5028.1	6817.8
ses	1			66.9	2459.7*	2921.5	3716.1	8834.8	97561.9
	230			208.8	477.9	793.4	1198.0	5974.4	7689.4
	4			285.3	1248.7*	1993.4*	3144.4*	25645.6**	32566.2**
	3			291.3	531.6	769.9	946.6	24342.5**	3203.6**
	7			261.9	878.0	698.0	879.7	3823.9	9796.3
	159			157.1	427.7	754.1	1182.2	4804.3	6734.8
es	1				18.8	2246.3	3322.1	2180.0	92216.1*
	230				260.5	599.5	867.4	5222.0	7277.9
	4				661.4*	1349.5	2025.0	19301.8**	26664.8**
	3				2120.4**	267.7	280.3	5206.3	3180.4**
	7				1775.7**	410.2	274.3	2141.8	8564.2
	159				234.8	645.4	882.1	4382.9	6501.1

l of 0.05 or lower.

l of 0.01 or lower.

Table 23. (continued)

Age (months)	Source		Degrees of freedom	Errors of estimate mean squares Ages (months) 1	2	3	4	5
4	Between Sindhi and crosses		1				1620.7*	283
	Remainder		230				239.5	55
	Proportions	(a)	4				3233.8**	27
	Breeds	(b)	3				4677.8**	6
	Interactions	(ab)	7				2499.5**	29
	Residual (error)		159				163.5	61
5	Between Sindhi and crosses		1					15
	Remainder		230					32
	Proportions	(a)	4					33
	Breeds	(b)	3					3
	Interactions	(ab)	7					52
	Residual (error)		159					34
6	Between Sindhi and crosses		1					
	Remainder		236					
	Proportions	(a)	4					
	Breeds	(b)	3					
	Interactions	(ab)	7					
	Residual		159					
12	Between Sindhi and crosses		1					
	Remainder		230					
	Proportions	(a)	4					
	Breeds	(b)	3					
	Interactions	(ab)	7					
	Residual (error)		159					

b.

ied)

	Degrees of freedom	Errors of estimate mean squares							
		Ages (months)							
		1	2	3	4	5	6	12	24
es	1					1620.7*	2837.7*	1892.1	82312.3
	230					239.5	556.9	4790.4	67082.0
	4					3233.8**	276.8	13385.8*	119318.1*
	3					4677.8**	68.0	10819.7*	495.3
	7					2499.5**	297.7	3653.6	9452.7
	159					163.9	616.0	4078.52	6122.9
es	1						156.3	24.9	53126.4**
	230						324.0	4520.1	6188.9
	4						330.5	12304.2*	15682.6*
	3						35.5	11535.7*	2854.2
	7						521.6	2831.3	6004.8
	159						348.3	3696.9	5646.9
es	1							544.4	51972.0**
	236							4869.1	6119.8
	4							8879.4*	16616.8*
	3							11676.7**	3694.3
	7							2590.1	8115.0
	159							2929.8	5520.4
es	1								75293.5**
	230								5000.7
	4								8224.0
	3								1478.7
	7								5508.9
	159								5127.1

DISCUSSION

The Data

One of the serious limitations of the data used in this study is the progressive reduction in the number of animals with advancing age. The total of 567 animals (365 females and 202 males) upon which at least one measurement of weight or height was taken compares favourably with other experiments on cross breeding. However, most such experiments naturally exhibit the same weakness as regards decreasing numbers with advancing age. This is illustrated by the statement of Morrison (1946, p. 614), who summarized data from several sources:

> For each breed, the averages represent data for a larger number of animals at early ages than at later ages.... The average heights shown in the table are for smaller numbers of animals and the number of bulls included in the studies was much less than heifers.

It is evident that the situation observed in these data is by no means peculiar to this particular study. In addition to the lower reliability which accompanies the reduction in number of animals, there is some danger of bias arising because of differential selection among the various groups carried on to older ages. Although this cannot be remedied now, it is to be considered in interpreting the data.

Males

The mean height for Sindhi male calves at birth was greater than that for cross bred calves (Tables 14 and 24). Since the difference was within the limits of expected probabilities (Table 15), no conclusion need be drawn from that single fact. The data on the height of male calves for older ages were too few to warrant the labour of detailed analysis. The mean heights of Sindhi and cross bred calves fluctuated from age to age, as is shown in Table 24, and at no age were the differences greater than those expected by chance variation.

Table 24. Mean heights and standard errors of Sindhi and cross bred males from birth to six months

Age (months)	Sindhi		Crosses	
	Mean (inches)	Number of observations	Mean (inches)	Number of observations
0	25.21±0.14	66	24.99±0.13	173
1	26.44±0.34	12	26.53±0.40	10
2	28.07±0.48	7	27.37±0.69	10
3	29.07±0.77	6	28.70±0.57	9
4	31.21±1.11	7	29.32±0.88	4
5	31.50±0.82	8	32.13±0.65	9
6	32.40±0.59	8	33.77±0.90	3

Because of the small numbers involved, and because the animals were the survivors of intense selection, the table is presented only to show that there was no consistent tendency for either group to excel in height at withers.

The difference in weight at birth of Sindhi and cross bred male calves (Table 14) was also of the same order as height. The observed difference easily could have occurred as a chance variation.

The constants for height and weight given in Table 14 are strikingly similar. Of the breeds studied, the calves with Brown Swiss breeding (b_2) were the largest at birth, both in height and in weight. The calves with Holstein breeding (b_3) were a close second in weight but they had the lowest development in height. The calves of Guernsey breeding (b_4) had the lowest birth weights but stood second in height. The calves with Jersey breeding (b_1) were intermediate. The constants indicate differences in body formation during prenatal growth. The Holstein (b_3) and the Guernsey (b_4) represent two opposite extremes in conformation as judged by height and weight at birth.

The effect of various proportions of European blood also demonstrates similar differences. In general, the values of constants a_4, a_5 and a_6 were negative for both height and weight. The positive value of the constant a_4 for the height is too small to have much meaning. The positive value of the constant a_6 for the weight is large enough not to be neglected and may represent a real tendency for the animals with 6/8 blood of European breeds to gain more weight during the prenatal period. These animals were produced by dams with 4/8 European blood which had been bred to pure bred bulls of European breeds. Since dams with 4/8 European blood grew to a larger size than others (Table 21), the heavier weight of the a_6 calves at birth may merely reflect superior maternal environment rather than a superior gene combination among the a_6 calves. This interpretation is also supported by the heavy weights of the a_2 calves which were produced by mating dams with 4/8 European blood to Sindhi bulls.

The results presented in Table 15 confirm the conclusion that the differences between Sindhi and cross bred animals are within the limits expected by chance variation. The differences attributable to the proportion of European blood and those between the four European breeds used were greater than expected at a probability level of one per cent.

Females

Height

The average height of Sindhi and cross bred females (Table 17, Figure 1) was similar from birth to 36 months of age. In general, the cross bred animals were born with slightly greater height than the Sindhi and maintained this lead throughout, except at 24 months of age. The differences between the two groups were no greater than those expected by chance at the one per cent level of probability. Figure 1 shows that the growth of Sindhi animals in height between two and three months was slow. This was made up by the time the calves were four months old. The difference between the mean heights of the Sindhi and the crosses at three months of age was greater than that expected at the five per cent level of probability but not greater than expected at the one per cent level. There seems to be no adequate explanation of the slower growth in height of Sindhi calves at three months. The records were made in different years at different periods of time and there does not seem to be any time trend in the data.

The cross bred animals averaged lower in height at 24 months than Sindhi animals. This may have been due to chance variation, since the difference was small and within the limits of expectations at the five per cent level of probability.

The main fact brought out by a study of the height data is that while the cross bred animals tend to have greater height, at no age is the difference from Sindhi animals greater than that expected due to chance variations at the one per cent level of probability. Hence one of the principal conclusions from these data is that the growth of Sindhi and the crosses in height is similar up to 36 months of age. This is supported by the results obtained in the analysis of variance (Table 19).

The values of various constants presented in Table 18 show that the females with Holstein breeding (b_3) had the greatest height at all ages studied. The animals with Brown Swiss breeding (b_2) were second in height. Animals with Jersey breeding (b_1) were slightly higher than those with Guernsey breeding (b_4) at birth, at 12 months, and at 36 months, while those with Guernsey breeding (b_4) were higher at six months and at 24 months. The data for animals with Guernsey breeding (b_4) were far from adequate (Table 16), and it does not seem worth while to pay more than casual attention to the values for this group.

The half bred animals (a_4) had low heights at birth but grew to be the highest at 12 months and later. Animals with 25 per cent

European blood (a_2) were born with the greatest height and kept this lead till six months, after which they stood second highest. The animals with 12.5 per cent European blood (a_1), stood second highest at birth but did not maintain this position after that. Animals in a_3, a_5 and a_6 groups were rather small at birth, those in group a_5 being the shortest.

The half bred (a_4) animals were obtained from pure bred Sindhi (a_0) dams. The one quarter (a_2) and one eighth (a_1) animals were obtained from half bred (a_4) and one quarter (a_2) Sindhi dams respectively. The small size of a_4 animals at birth may be accounted for by the maternal environment provided by their Sindhi dams, whereas the large size of a_2 calves is explained by their having half bred dams. The height at birth of the calves is in exactly the same order as the height of the dams at 36 months. As the animals grew older, the influence of maternal environment was progressively obliterated. This is illustrated in Figure 3.

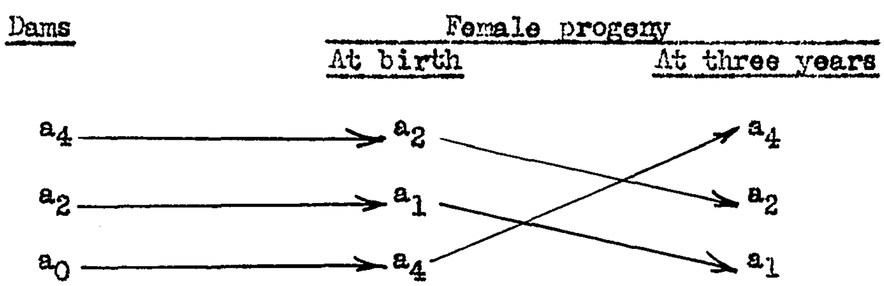

Figure 3. Rank in height at withers of dams and of their female progeny with different proportions of European blood.

These results are strikingly similar to those reported by Walton and Hammond (1938) in a study of reciprocal crosses of Shire horses and Shetland ponies. These authors found that the size of the foals at birth was approximately proportional to the weight and height of their dams and about equal to the foals of the pure breeds to which their dams belonged.

The analysis of variance presented in Table 19 shows that at none of the ages studied were the differences in the height at withers between Sindhi and the cross bred animals more than expected by chance variations in the data. The differences in the height at birth attributable to different proportions of European blood were greater than expected due to chance variation at the one per cent level of probability, while the breeds of European cattle differed significantly at birth and at 36 months. The differences at birth reflect the differences both in the intra-uterine environment and in the inherent growth rate, while those at later ages show more clearly the influences of external environments and the inherent growth rate. Later in this study, the theme regarding the importance of maternal environment in relation to size at birth is developed more fully. The management practices were reasonably uniform for all animals. It can, therefore, be assumed that external environments did not tend to favour any one group more than any other. The large mean square for breeds at 36 months (Table 19) probably represents the differences in the age at which different breeds reach maturity. The rank of the breeds at 36 months was Holstein-Friesian, Brown Swiss, Jersey and Guernsey, with the Jersey and Guernsey very close together. Except for the last pair, the mature size of the pure breeds under their natural environments is in the same order.

The fact that the mean squares for the interaction of breed and blood percentage (ab) were small (Table 19), suggests that the effect of changing the amount of European blood was similar for each of the four breeds studied.

Weight

The female cross bred calves averaged heavier at birth than the Sindhi calves and became increasingly heavier with advancing age (Table 20). These differences were greater than can be ascribed to chance variation. In this respect data for weight differ from the data for height. This is in conformity with the findings of Swett *et al.* (1929), who studied a Jersey and an Aberdeen Angus cow as representatives of dairy and beef types. They concluded that the skeletal development of the two was generally similar and that the difference in conformation between the two was caused by the development of extreme "fleshing" in the Aberdeen Angus cow.

It is obvious from the magnitude of differences that the Sindhi and the crosses had different rates of growth. Blackman (1919) proposed a measure of growth rate in plants. He called it "Efficiency index". Fisher (1921) termed this "Relative growth rate" and reiterated that this was a correct measure of growth. Brody (1945) has used this statistic, called K, as a measure of growth rate. It has been defined as,

$$K = \frac{l_n W_1 - l_n W_2}{t_1 - t_2}$$

Where, $l_n W_1$ and $l_n w_2$ are the natural logarithms of weights, and

t_1 and t_2 are the corresponding time units defining the time interval between the two observations.

When multiplied by 100, the K statistic describes the percentage growth rate. In order to study the comparative growth rates of Sindhi and crosses, the percentage growth rates of the two groups have been presented in Table 25 and illustrated in Figure 4.

Table 25. Percentage growth rate in the weight of Sindhi and cross bred calves from birth to two years

Age (months)	Sindhi	Crosses
0- 1	41.4	38.7
1- 2	25.5	28.4
2- 3	23.5	25.7
3- 4	22.1	20.7
4- 5	15.5	17.8
5- 6	14.0	15.6
5-12	9.5	9.0
12-24	2.8	3.4

The growth rate of crosses is higher than that of Sindhi animals at all age intervals except between birth and one month and between six and 12 months. The lower growth rate between birth and one month is

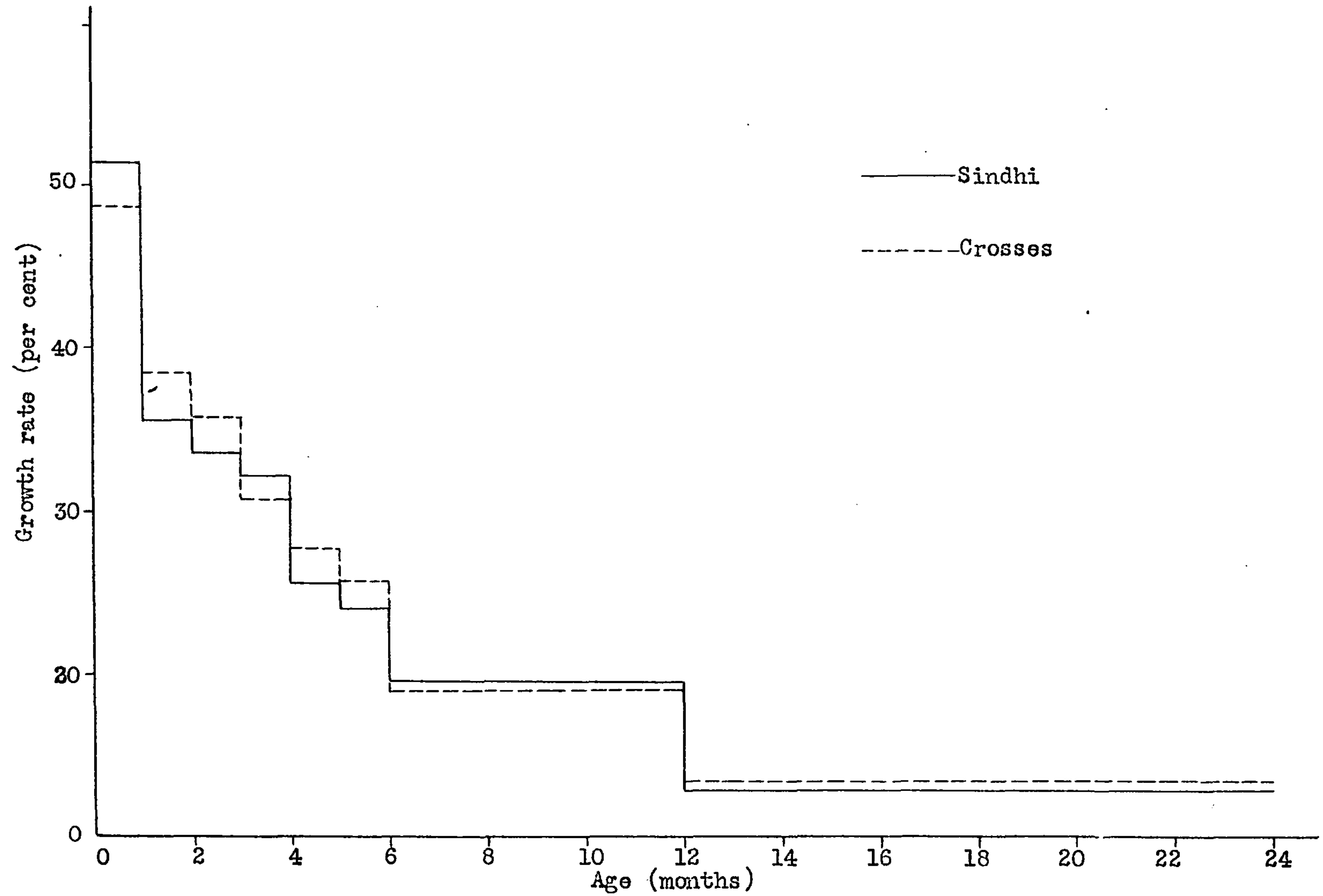

Figure 4. Relative percentage growth rate in weight of Sindhi and cross bred females.

probably indicative of the comparative inability or rather slowness of the cross bred calves to adjust themselves to the external environments. In a similar manner the lower growth rate between six and 12 months may be due to the inability of these calves to adapt themselves to changes in feeding consequent to the discontinuance of skimmilk in the feed. The simplest interpretation of these results is that the cross bred calves are not able to adjust themselves to changes in environments as quickly as Sindhi calves. This is in confirmity with the general opinion that cross bred cattle need greater care and attention in the Tropics than do animals of indigenous breeds.

The constants shown in Table 21 present a picture similar to those for height. The animals with Brown Swiss breeding (b_2), heaviest at birth, surrendered this position to animals with Holstein breeding (b_3) at the later ages. The animals with Jersey breeding (b_1) had the lowest weight at birth and at 24 months, while those with Guernsey breeding (b_4) occupied this position from one to 12 months of age. The large

Table 26. Weights of animals having different breeding

Age (months)	Sindhi (mean)	Cross bred Jersey ($\mu + b_1$)	Cross bred Brown Swiss ($\mu + b_2$)	Cross bred Holstein-Friesian ($\mu + b_3$)	Cross bred Guernsey ($\mu + b_4$)
0	42.5	38.3	47.6	44.0	45.5
1	64.3	58.8	65.1	66.0	53.5
2	83.0	81.3	88.7	98.2	80.5
3	104.9	107.2	113.6	124.4	94.9
4	130.7	135.3	147.2	156.3	123.3
5	152.8	162.4	173.7	181.6	144.6
6	175.6	192.4	205.5	211.7	173.3
12	312.8	330.1	342.4	388.3	285.9
24	446.6	485.8	510.3	538.8	506.1

negative constant for animals with Jersey breeding (b_1) at 24 months probably reflects the early maturing tendency of this breed of cattle. The average weights of animals with different breeding as determined by the fitted constants, are presented in Table 26.

The weights of the animals with Jersey breeding (b_1) were closest to those of Sindhi animals. The animals with Guernsey breeding (b_4) made slow gains. They were born heavier than the Sindhi calves but lagged behind till 24 months of age, when they were again heavier than Sindhi animals. These lower gains may represent the inability of these animals to make sufficient growth in the environments provided. This may be interpreted as an expression of the poor adaptability of these animals to tropical conditions. This belief is reinforced by findings of Cousins (1933) who reported that Guernsey cattle were unsuitable as a pure breed in Jamaica. In a study of crosses of three European breeds with Sahiwal cattle in Jamaica, Howe (1946, p. 86) stated:

Table 27. Relative per cent growth rate of Sindhi and cross bred females with different breeding

Age (months)	Per cent growth rate (100 K)				
	Sindhi	Cross bred			
		Jersey	Brown Swiss	Holstein-Friesian	Guernsey
0- 1	41.4	41.7	31.4	41.3	23.7
1- 2	25.5	31.5	31.7	40.0	34.3
2- 3	23.5	29.5	24.3	22.1	15.1
3- 4	22.1	23.1	27.5	23.4	26.0
4- 5	15.5	18.9	16.0	15.0	15.0
5- 6	14.0	16.0	17.5	15.1	19.0
6-12	9.5	9.1	8.6	10.0	8.3
12-24	2.8	3.3	3.4	2.6	4.8

The manner in which the Guernsey breed varies in its reaction with Zebu blood makes it doubtful whether the crossing of this breed with Zebu would be advantageous in developing a new breed for conditions in Jamaica.

Table 27 confirms the erratic behaviour of cross bred female calves with Guernsey breeding. It also illustrates that the growth rate of Jersey crosses was nearest to that of pure bred Sindhi animals.

On the basis of these data, it can be said that if a Sindhi animal was the ideal in size and if a cross was desired that would have the least difference from it, in this respect, the Jersey, of the European breeds studied, came closest to meeting this requirement. On the other hand, if the largest animal is considered most desirable, animals with Holstein breeding would be most suitable among the breeds studied.

The influence of varying proportions of European blood on the growth in weight was approximately parallel to that on the growth in height. The influence of maternal environments is demonstrated by the growth of animals in a_4, a_2 and a_1 groups. In general, the birth weights of calves were in the same order as the weights of their dams at two years (and probably at maturity). The weights of groups a_5 and a_6 do not conform to this. In the data for animals weighed at each age (Table 21), the a_5 and a_6 groups have the lowest birth weights. The constants for all data at birth (page 34), show that the birth weight of the a_6 group was second highest, but the group a_5 had the lowest birth weight in these data. This was also true of the birth weights of male calves (Table 14). The rank of dams at two years and of calves at birth having different proportions of blood of European cattle is illustrated in Figure 5.

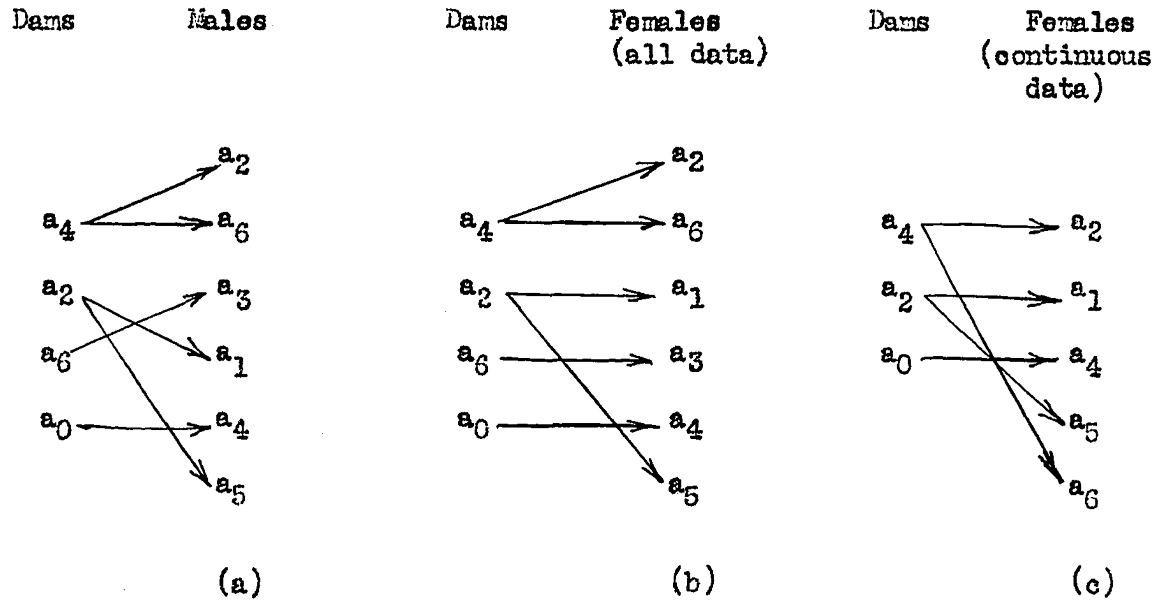

Figure 5. Ranks in weight of dams at two years and of the progeny at birth.

Figures 5(a) and 5(b) are quite similar, the only difference being an interchange in the position of the a_3 and a_1 calves. The difference in the position of a_6 between Figures 5(b) and 5(c) is accounted for by the non-inclusion of the birth weight of four calves of the group b_3a_6 (6/8 Holstein-Friesian) in the study of continuous data presented in Table 21. These calves had heavy birth weights averaging over 61 pounds and almost entirely account for raising the constant for the a_6 group in the data presented on page 34. Subsequent readings on these animals were not available and hence they are not included in the analysis of continuous data. The resemblance between Figures 5(a) and 5(b), both of which are based on the large amount of unselected data confirms the conclusion that the birth weights are influenced by the maternal environments. It also shows that there is no sex difference in this respect.

The constants (Table 21) show that the animals with 50 per cent European blood (a_4) were not heaviest at birth but at the age of four months they had grown to have the highest weight and they maintained this lead for the rest of the period studied. Animals with 25 per cent European blood (a_2) were born heaviest and had the highest weight for the first two months. From three months to 24 months these animals were second highest in weight, except at 12 months when animals of the a_5 group exceeded them. Animals with 12.5 per cent European blood (a_1) were born second heaviest but did not maintain this position after they were one month old. The growth of animals with 62.5 per cent European blood (a_5) was rather erratic. They were born with the second lowest weight but fluctuated much and occupied the third highest position at 24 months. The constants demonstrate clearly unthriftiness (and therefore unsuitability) of animals with 75 per cent blood of European breeds (a_6). These animals had lowest weights at all ages studied, except at six months when they were second lowest. Animals with more than 75 per cent European blood were not available; but on the basis of these data, it seems reasonable to conclude that animals with 75 per cent or more of European breeds do not grow so well as those with lower percentages do. If large gains in the weight are desirable, these animals do not measure high on the scale of desirability.

The animals of the a_4 group were obtained by using purebred bulls of European breeds (a_8) on Sindhi (a_0) cows. These animals were born small, probably because of the limitations of their Sindhi dams to provide intra-uterine environments that would give fair expression to their inherited growing ability. The fact that these animals overcame the handicap of lower birth weight and grew to be the heaviest among the crosses gives some indication of heterosis.

Four variables are involved in the preceding discussion, namely: (1) percentage of Sindhi blood (S), (2) percentage of European breed (E); (3) percentage heterozygosity of progeny (P); and (4) percentage heterozygosity of dam (D). Numerical values, which demonstrate the relative importance of a unit change in each of these variables upon the weight of the cross bred calves, can be computed from the "a" values given in Table 21. Each "a" can be expressed as a linear function of the four variables enumerated above. Thus,

$$a_i = C_s S_i + C_e E_i + C_p P_i + C_d D_i,$$

Where the C's are constants whose values are to be determined. According to this scheme, five equations can be written to describe the five "a" constants of Table 21:

$$a_1 = 87.5C_s + 12.5\ C_e + 25\ C_p + 50\ C_d$$

$$a_2 = 75.0C_s + 25.0\ C_e + 50\ C_p + 100\ C_d$$

$$a_4 = 50.0\ C_s + 50.0\ C_e + 100\ C_p + 0\ C_d$$

$$a_5 = 37.5C_s + 62.5\ C_e + 75\ C_p + 50\ C_d$$

$$a_6 = 25.0C_s + 75.0\ C_e + 50\ C_p + 100\ C_d$$

One of the equations can be eliminated, or two can be combined, and the remaining four solved simultaneously to obtain values of the C's. The a_6 equation was dropped in the solution given here because only four animals were in that class.

Table 28. Values of variables for percentage of blood and heterozygosis in the weight of females

Variables	Age (months)			
	0	6	12	24
C_s	+0.013	-0.344	-0.777	-0.816
C_e	-0.291	-0.582	-0.988	-2.266
C_p	+0.123	+0.596	+1.340	+2.097
C_d	+0.095	+0.170	+0.252	+0.432

The constants at ages 0, 6, 12 and 24 months are given in Table 28. Caution must be used in interpreting these constants as absolute values for several reasons. First, the "a" effects were originally estimated with the restriction that $\Sigma a_i = 0$. Second, the sum of the four equations actually solved is

$$250\ C_s + 150\ C_e + 250\ C_p + 200\ C_d = a_1 + a_2 + a_4 + a_5 = -a_6.$$

Third, an increase or decrease in Sindhi blood is necessarily accompanied (biologically) by a compensating change in European breeding. The following conclusions are suggested from observing the relative magnitude of the constants:

(1) Increasing the percentage of Sindhi blood at the expense of European blood results in increased growth.

(2) The detrimental effect of European blood becomes more pronounced with increasing age, suggesting lack of adaptability.

(3) Heterozygosity of the calf is accompanied by increased growth, apparently more pronounced at older ages.

(4) Heterozygosity of the dam is accompanied by increased weight of the calf, particularly at birth.

Findings of Howe (1946) also indicate that heterozygosis was a strong influence in the growth of cross bred animals. Recognizing that Howe's study was conducted under different environmental conditions (in Jamaica) and that a different breed of Indian cattle (Sahiwal) was used, the comparison is valid only in general terms. In the data presented by Howe (1946, p. 46) the a_4 group had the highest weights at all ages up to 13 years. The pure bred animals of European breeds (a_8) had the lowest weight at birth and had generally low weights at all other ages (up to 13 years). Howe felt that his data did not permit separating the effect of percentage of blood from the effect of heterosis and attributed the higher weight and the growth of the half bred animals to their increased percentage of Sahiwal blood. In view of the fact that he did not have animals with Sahiwal blood greater than 50 per cent nor did he have half bred animals produced in ways other than by crossing pure breeds, the conclusion was justified. In the present study an effort to measure these factors separately has given entirely reasonable results. The method is not necessarily limited to the linear model used here although the adequacy and range of the present data discourage further extrapolation. For example, the equations could have been written as quadratics in one variable by including the a_6 equation in the preceding analysis.

The mean squares in the analyses of variance (Table 22) confirm the finding that the observed difference between the weight of Sindhi and that of the cross-bred animals is greater than would be expected by chance. These also show that the differences attributable to proportions of European blood are generally greater than those expected at the one per cent level of probability. The same is true of differences between the four breeds of European cattle. These results show that the weights of animals at the ages studied were significantly influenced by the breed-blood percentage group to which they belonged. In practice, this makes it necessary to pay attention to the particular breed and the proportion of blood used for cross breeding. The non-significance of mean squares for interaction between proportions of blood and breeds (ab) indicates that the non-additive relationships between the different breeds and varying proportions of European blood were unimportantly small, although of course we cannot be sure they were zero.

The analyses of covariance between the weights at different ages were carried out. The mean squares in errors of estimate for the weights at later ages (dependent variables) from a statistical control of the weights at earlier ages (independent variables) are given in Table 23. These show the following features:

a. The mean squares for the errors of estimate between Sindhi and the crosses are generally greater than expected at the five per cent level of probabilities but not greater than those expected at the one per cent level. This shows that the difference between the weights of Sindhi and crosses at the earlier ages are only slightly valuable for predicting this difference at the later ages.

b. The error mean squares between the proportions of European blood (a) are, in the main, greater than expected because of chance variation at the one per cent level of probability. The early differences in the weight associated with the various percentages of European blood do not persist strongly into the later ages. Although this pattern is by no means perfect, the tendency is clearly discernible. The most logical genetic explanation seems to be that different sets of genetic influences cause these differences early and later in the growing period.

c. The error mean squares in the analysis of covariance, attributable to differences between the four European breeds (b) are more irregular than in the preceding case for the various percentages of blood. The regressions from 1 to 6 months are quite effective in predicting the weight of the calves during the latter part of the period studied. The differences attributable to the four European breeds fluctuate when the calves are young but there is a tendency for these differences to follow a set pattern when the animals are older. This conclusion must remain in general terms because of several exceptions.

d. In the covariance analyses, the interaction (ab) covariances at most of the ages studied were small but positive. This indicates that small plus or minus deviations from the general trend of breeds and percentages of blood at early ages persist until later ages. This is an entirely reasonable result since it is to be expected that growth at any period depends upon the growth at earlier periods, regardless of the causes of the earlier growth.

The errors of estimate for the interaction term in the covariance analyses were notable only for their small size. Nearly all of them were less than would be expected by chance variation at five per cent level of probability and several of them were less than the residual mean squares which have been used as error terms. This shows that the interaction terms at later ages were almost entirely determined by the regression on the weight at earlier ages.

The results obtained in this study are limited by insufficient data, as regards height at withers. The data on crosses involving the Guernsey breed and those having more than 50 per cent blood of European cattle were also small in amount. More records of the height can probably be obtained without much additional expense and efforts. But obtaining additional crosses to supplement the data on the animals with Guernsey breeding and on animals with more than 50 per cent of the blood of European breeds is in a different category. Both are matters primarily concerned with the economics of running the herd. In a college herd, such as that at Allahabad, not primarily devoted to research, the small additional expense in obtaining height records may be justified but obtaining additional crosses may not be. In any case it would be difficult because the herd has no Guernsey bull at present and the policy of back crossing to the Sindhi parent has gone quite far.

The decision to emphasize the breeding of the Jersey in preference to the other breeds of European cattle was a sound one if cross bred animals with the least difference from Sindhi animals in the weight and the height were desired. The size of indigenous animals in any partic-ular set of environments has been a result of adjustments to the pre-vailing nutritional and climatic conditions. Any wide deviation from it would be likely to add to the unsuitability of the animals and efforts to modify environments would result in expense which may not be economically justifiable. The Sindhi animals have shown themselves to be well adapted to the local environments and the Jersey crosses were closest to the Sindhi animals in size. While recognizing that size should not be the primary or sole aim in breeding of dairy cattle, the similarity of the Jersey crosses to Sindhi animals in this respect adds a point in their favour.

CONCLUSIONS

1. The differences in height between the Sindhi and the average of all cross bred females from birth to three years of age were not large enough to be significant.
2. The female cross bred calves with Holstein-Friesian breeding had the greatest height at birth and remained so at all ages studied (up to three years). Those with Guernsey breeding had the least height at birth and retained this position at all ages studied except at two years.
3. Cross bred female calves having 50 per cent of the blood of European cattle were not born with the greatest height, but attained this position at one year and maintained it for the rest of the period studied. Those with 25 per cent of the blood of European cattle had the greatest height at birth but occupied the second highest position at one year and later.
4. Differences in the height of cross bred female calves, attributable to different proportions of blood of European cattle, were not significant at any age except at birth. Those attributable to the breed of European cattle were significant at no age except at birth and at three years. The significant difference at three years was apparently indicative of a tendency for some breeds to reach skeletal maturity earlier than others.

5. The difference between the weight of Sindhi and that of cross bred female calves was non-significant at one, four and 12 months. It was significant at the five per cent level at birth and at two and three months, and at the one per cent level at five, six and 24 months.

6. The growth rate of cross bred females was higher than that of Sindhi at all ages studied except between birth and one month and between six and 12 months. Since these periods were characterized by change in environments, the lower growth rate indicates that the cross bred females had less adaptability to changes in the environments than the Sindhi animals.

7. Cross bred animals with Brown Swiss breeding were heaviest at birth but those with Holstein-Friesian breeding were heaviest at two years. The growth of cross bred calves with Guernsey breeding was slowest of all the different crosses studied. These calves also appeared to respond adversely to unfavorable changes in environment more noticeably than the other breeds. The weights of the cross bred animals with Jersey breeding were closest to those of Sindhi animals.

8. The cross bred calves with 50 per cent blood of European breeds did not have the highest weight at birth, but they gained rapidly and had the highest weight at two years. The weights attained by those with 75 per cent blood of European breeds were lower than those having a lower percentage of European blood, thus indicating that these animals were not suitable under the environments provided.

9. The differences in the weights of cross bred calves due to the varying proportions of blood of European cattle were non-significant at two months. They were significant at the five per cent level at three and six months and at the one per cent level at birth and at one, four, five, 12 and 24 months. The differences between the breeds of European cattle were non-significant at the five per cent level at five and six months and at the one per cent level at other ages.

10. The absence of interactions, between the breeds of European cattle and the proportions of their blood used in cross breeding, for the height as well as for the weight indicated that a similar response resulted from increasing the percentage of blood of each of the breeds.

11. The weights and heights of cross bred calves at birth indicated strong influence of maternal environments. This influence gradually diminished as the animals grew older.

12. The increase in the percentage of Sindhi blood at the expense of the blood of European breeds increased growth.

13. There were indications that the detrimental effect of European blood on growth rate became more pronounced as the animals grew older; suggesting lack of adaptability.

14. The heterozygosis of the calf was accompanied by increased growth, particularly at older ages, while the heterozygosis of the dam accompanied increased weight of the calf, especially at birth.

SUMMARY

The growth of Sindhi and crosses of Sindhi with Jersey, Brown Swiss, Holstein-Friesian and Guernsey breeds of cattle has been studied. The data were obtained from Allahabad Agricultural Institute, Allahabad, India. The herd at Allahabad was established in 1910. Several breeds of Indian cattle were tried and eventually animals of the Sindhi breed were kept. Cross breeding with European breeds was started in 1922 and the four European breeds of cattle mentioned were used. In 1934, a policy of back crossing the cross bred animals to the Sindhi was initiated and emphasis was placed on breeding Sindhi animals with Jersey.

The records studied consisted of observations on weight and height at monthly intervals from birth to six months, then at one year and annually thereafter. A few records of birth weights were available from 1928, but the bulk of the data for the weights were obtained from 1934 to 1948. Records for height were available from the end of 1935 to 1948. There were more data for weight than for height, for early ages than for later ages, and for females than for males.

The data were grouped according to the breeds and according to the proportions of blood of European breeds in units of 1/8. Animals with 5/8 blood of European breeds were represented by those with Jersey breeding only. The crosses with Guernsey breeding were represented in 1/8, 2/8 and 4/8 blood groups only.

Analyses of variance and covariance were carried out. Because the data were disproportional in various classifications and because some classes were missing altogether, the least squares estimates were obtained by the method of fitting constants. The method has been described in detail.

The data for the height and the weight of male calves were studied for observations at birth only. The Sindhi male calves averaged 25.21 ± 0.14 inches in height at birth (66 observations) and their mean birth weight was 44.5 ± 0.9 pounds (72 observations). The height of cross bred male calves at birth was 24.99 ± 0.13 inches (173 observations) and their birth weight was 46.0 ± 0.8 pounds (202 observations). The differences between Sindhi and cross bred males in height and weight at birth were not significant. Among the cross bred male animals, the differences attributable to the four breeds of European cattle and to the proportion of European blood were significant at the one per cent level. The differences due to interactions between these two variables were not significant.

The mean heights of Sindhi and cross bred female calves from birth to three years are presented. The average wither height of female Sindhi calves was 24.85 ± 0.15 inches (103 observations) at birth and 43.94 ± 0.37 inches (18 observations) at three years. The height of female cross bred calves was 24.94 ± 0.10 inches (298 observations) at birth and 44.41 ± 0.21 inches (67 observations) at three years. The differences between the two groups were not significant at any age.

Analyses of variance for height at birth, at six, 12, 24 and 36 months was carried out. The differences attributable to the breeds of European cattle were significant (one per cent level) at birth and at three years, while the differences due to proportions of blood of European breeds were significant (one per cent level) at birth only.

The average weight of Sindhi females was 42.5 ± 0.8 pounds at birth (115 observations) and 446.6 ± 12.6 pounds at two years (58 observations). The mean weights of cross bred females were 44.7 ± 0.6 pounds (365 observations) and 500.5 ± 6.8 pounds (194 observations) respectively at birth and at two years.

Analyses of variance for weights at each age observed were carried out for 233 females who had continuous records from birth to two years. The mean weights of Sindhi and cross bred female calves were significantly different at all ages studied except at one and four months. The level of significance exceeded was five per cent at birth, at two and three months and one per cent at 5, 6, 12 and 24 months.

The differences in the weight of cross bred female calves, attributable to the varying proportions of blood of European breeds, were not significant at two months. They were significant at the five per cent level at three and six months and at the one per cent level at other ages. The differences due to breeds of European cattle were not significant at 24 months; but were significant at the five per cent level at five and six months and at the one per cent level at birth, at one, two, three, four, 12 and 24 months. The differences due to interactions between these two variables were not significant at any age.

Analyses of covariance between weights of the female calves at different ages were carried out. The weights at earlier ages were considered independent variables. The errors of estimate from regressions of weights at later ages on weights at earlier ages have been presented. The results showed that the mean squares in the errors of estimate between Sindhi and crosses were generally significant at the five per cent level of probabilities.

In the data for crosses, the error mean squares for differences between proportions of blood of European cattle were generally significant at the one per cent level of probabilities. This indicated that a knowledge of such differences at earlier ages was not valuable in predicting such differences at later ages.

The differences due to breeds tended to follow the pattern of differences at earlier ages when the animals were older but not when they were young. The differences due to interactions between these two variables generally followed the pattern of such differences at the early ages.

The lack of significant interactions indicated that the genetic factors motivating growth in height and weight were additive in nature.

The data on the height and the weight showed that the Guernsey crosses did not grow so well as others. The Holstein-Friesian crosses grew to be the largest animals. Of the four types of cross breds studied, the growth of Jersey crosses was most similar to that of Sindhi animals.

The data also showed that the animals with 50 per cent European blood had the fastest growth and that animals with 75 per cent blood of European breeds were generally unthrifty.

There was some evidence that the influence of maternal environment was strong at birth, but wore out as animals grew older. The data also showed that the heterosis of dam was accompanied by heavy weights of calves at birth; but the heterosis of calf was associated with greater gains in weight at later ages.

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